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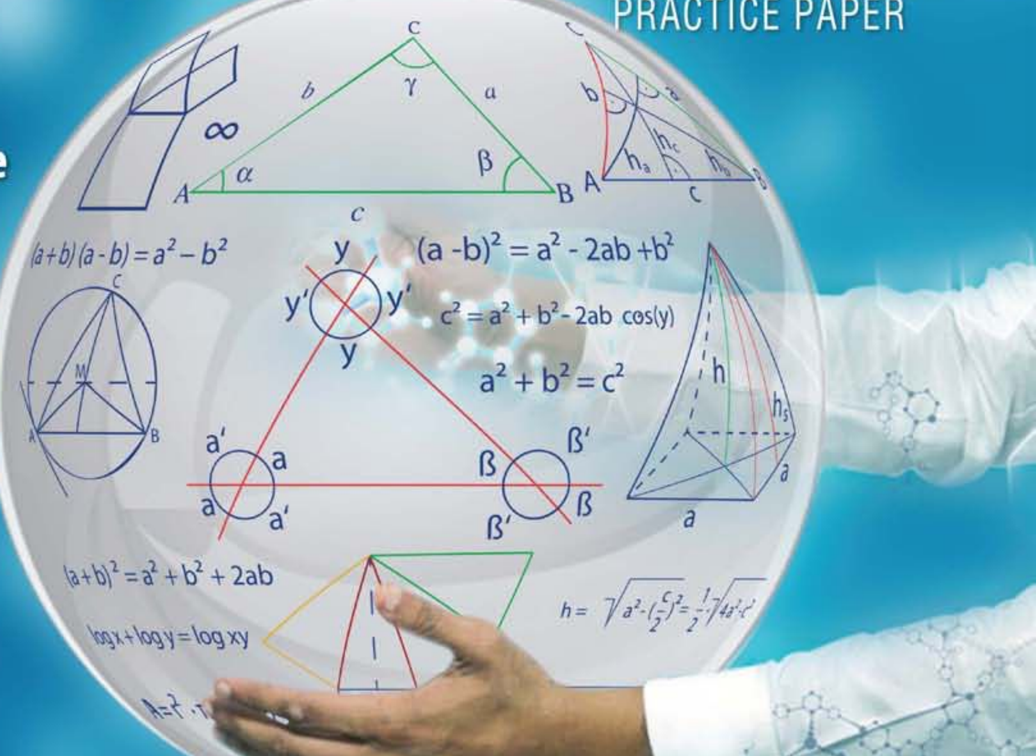


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# MATHEMATICS today

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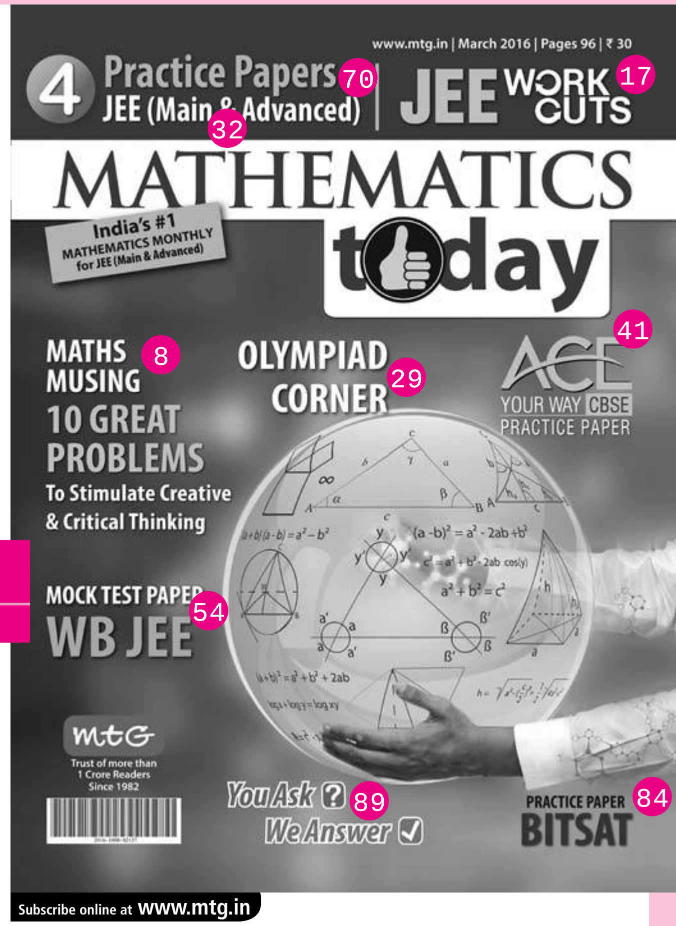





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Editor : Anil Ahlawat

# MATHS MUSING

Maths Musing was started in January 2003 issue of Mathematics Today with the suggestion of Shri Mahabir Singh. The aim of Maths Musing is to augment the chances of bright students seeking admission into IITs with additional study material.

*During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.*

*Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.*

## PROBLEM Set 159

### JEE MAIN

**1.** Let $S = \{2^0, 2^1, 2^2, \ldots, 2^{10}\}$. Consider all possible positive differences of elements of S. If $M$ is the sum of all these differences, then the sum of the digits of $M$ is

(a) 24 (b) 27 (c) 30 (d) 31

**2.** A fair coin is tossed 12 times. The probability of getting atleast 8 consecutive heads is

(a) $\frac{3}{2^7}$ (b) $\frac{5}{2^7}$ (c) $\frac{5}{2^8}$ (d) $\frac{3}{2^8}$

**3.** If $\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\frac{1}{3\cdot4\cdot5}+\ldots+\frac{1}{10\cdot11\cdot12}=\frac{m}{n}$, reduced fraction, then the sum of the digits of $(m + n)$ is

(a) 13 (b) 14
(c) 15 (d) 16

**4.** If $\alpha$, $\beta$ are the roots of $x^2 + ax - \frac{1}{2a^2} = 0$, $a \neq 0$, then the minimum value of $\alpha^4 + \beta^4$ is

(a) 2 (b) $3+\sqrt{2}$
(c) $2+\sqrt{2}$ (d) $4-\sqrt{2}$

**5.** If $0<c<1$ and $\int_0^{\pi/2} \sin^{-1}(c\cos x)dx = \frac{c}{a_1}+\frac{c^3}{a_2}+\frac{c^5}{a_3}+\ldots$, then $(a_1 + a_2 + a_3)$ is divisible by

(a) 3 (b) 5 (c) 7 (d) 11

### JEE ADVANCED

**6.** Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of a $\Delta ABC$. A parallelogram $AFDE$ is drawn with $D$, $E$ and $F$ on the line segment $BC$, $CA$ and $AB$ respectively. Then, maximum area of such a parallelogram is

(a) $\frac{1}{2}(\text{area of } \Delta ABC)$ (b) $\frac{1}{4}(\text{area of } \Delta ABC)$

(c) $\frac{1}{6}(\text{area of } \Delta ABC)$ (d) $\frac{1}{8}(\text{area of } \Delta ABC)$

### COMPREHENSION

Consider the line $\vec{r} = -\hat{j}+\hat{k}+\lambda(-2\hat{i}+2\hat{j}+\hat{k})$, and the points $C(1, 2, 3)$ and $D(2, 0, 0)$

**7.** The distance of point $D$ from the plane through the line and the point $C$ is

(a) $\sqrt{3}$ (b) $\sqrt{\frac{5}{2}}$
(c) $\sqrt{7}$ (d) $\sqrt{10}$

**8.** The distance of the point $D$ from the image of the point $C$ in the line is

(a) $\sqrt{15}$ (b) $\sqrt{20}$
(c) $\sqrt{29}$ (d) $\sqrt{34}$

### INTEGER MATCH

**9.** The values $15x = 8y$ and $3x = 10y$ contain points $P$ and $Q$ respectively. If the midpoint of $PQ$ is (8, 6), then the length of $PQ = \frac{m}{n}$ reduced fraction, where $(m - 8n)$ is

### MATRIX MATCH

**10.** Let $S = \{1, 2, 3, \ldots, 10\}$

| | Column I | | Column II |
|---|---|---|---|
| (a) | The number of subsets $\{x, y, z\}$ of $S$ so that $x, y, z$ are in A.P. | (p) | 15 |
| (b) | The number of subsets $\{x, y, z\}$ of $S$ so that no two of them are consecutive. | (q) | 20 |
| (c) | The number of subsets $\{x, y\}$ of $S$ so that $x^3 + y^3$ is divisible by 3. | (r) | 24 |
| (d) | The number of subsets $\{x, y\}$ of $S$ so that $x^2 - y^2$ is divisible by 3. | (s) | 30 |
| | | (t) | 56 |

See Solution set of Maths Musing 158 on page no. 82

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**К сожалению, в последнее время нашими трудами пользуются другие группы: более раскрученные, но, почему-то, не желающие самостоятельно искать для себя контент.**

**Поэтому если Вы скачали этот журнал в другой группе, просим Вас лишь об одном: зайдите в нашу группу и, если понравится, подпишитесь. Поддержите наш труд, а мы каждый день будем радовать Вас новыми и интересными журналами на английском языке. Спасибо!**

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# PRACTICE PAPER 2016
# JEE MAIN

1. If α, β are the roots of the equation $x^2 - 2px + 2p^4 = 0$ $(p \in R)$, then maximum value of $\alpha^2 + \beta^2 =$
   (a) $-1$ (b) $1$ (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

2. The plane $\alpha y + \beta z = 0$ is rotated through an angle θ about its line of intersection with the plane $x = 0$, then equation of the plane in new position is
   (a) $\alpha y + \beta z \pm \sqrt{\alpha^2 + \beta^2} \sin\theta x = 0$
   (b) $\alpha y + \beta z \pm \sqrt{\alpha^2 + \beta^2} \cos\theta x = 0$
   (c) $\alpha y + \beta z \pm \sqrt{\alpha^2 + \beta^2} \tan\theta x = 0$
   (d) $\alpha y + \beta z \pm \sqrt{\alpha^2 + \beta^2} \sec\theta x = 0$

3. The range of $x$ so that in the expansion of $(1 + x)^{12}$, the numerically greatest term has the greatest coefficient is
   (a) $\left(\frac{-7}{6}, \frac{-6}{7}\right) \cup \left(\frac{6}{7}, \frac{7}{6}\right)$ (b) $\left(\frac{-8}{7}, \frac{-7}{8}\right) \cup \left(\frac{7}{8}, \frac{8}{7}\right)$
   (c) $\left(\frac{-6}{5}, \frac{-5}{6}\right) \cup \left(\frac{5}{6}, \frac{6}{5}\right)$ (d) $\left(\frac{-9}{8}, \frac{-8}{9}\right) \cup \left(\frac{8}{9}, \frac{9}{8}\right)$

4. The value of $\lim_{x \to 0} \frac{\cos(\tan x) - \cos x}{x^4} =$
   (a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

5. The value of $\int_{\ln\frac{1}{2}}^{\ln 2} \frac{e^{\sin x}}{e^{\sin x} + e^{-\sin x}} dx$
   (a) ln 8 (b) ln 4 (c) ln 2 (d) – ln 2

6. If $x$ is such that $[2 \sin x] + [\cos x] = -3$, for some $x \in [0, 25]$ then the range of the function $g(x) = \sin x + \cos x$, where $[\cdot]$ represent greatest integer function, is
   (a) $\left[-\sqrt{2}, -1\right]$ (b) $\left[-\sqrt{2}, -1\right)$
   (c) $\left(-\sqrt{2}, \frac{-1-\sqrt{3}}{2}\right)$ (d) $\left[\frac{-1-\sqrt{3}}{2}, -1\right]$

7. If $f(x) = \{x\} + \{-x\}$ & $g(x) = \sin^{-1} x$, then number of elements in the range of $gof(x)$ is (where $\{\cdot\}$ represents fractional part function)
   (a) 0 (b) 1 (c) 2 (d) 3

8. The number of real solution of $(x, y)$ where $|y| = \cos x$ & $y = \cos^{-1}(\cos x)$, $-2\pi \le x \le 2\pi$ is/are
   (a) 1 (b) 2 (c) 3 (d) 4

9. Let $f(x) = [x + 2]$ and $g(x) = \cos x$, where $[\cdot]$ denotes the greatest integer function. Then the value of $(fog)'(\pi/2)$, is
   (a) 0 (b) 1
   (c) –1 (d) Does not exist

10. Number of points of non-differentiability of $f(x) = \min\{0, \cos x, \sin x\}$ in $(0, 2n\pi)$ is
    (a) $3n - 1$ (b) $3n$ (c) $3n + 1$ (d) $3n + 2$

11. If the principle argument of a complex number $(-1 + i)^{50}$ is $\frac{n\pi}{2}$ and the equation $z^2 - az + b + 2i = 0$ has one real root. If $a = (1 + i)^{-n}$ and $b \in R$, then other root of given equation is
    (a) 1 (b) – 1 (c) $1 + i$ (d) $-1 + i$

12. If the equation $f(x) = x^3 + 3x^2 - 9x + c = 0$, $c \in R$, has two equal roots and one distinct root, then the total number of values of $c$ is
    (a) 0 (b) 1 (c) 2 (d) 3

13. Let $a, b, c$ be in G.P., $a, b, c, \in C - \{0\}$, *i.e.*, set of complex numbers except zero and $a \ne b \ne c$. If $a + b$, $b + c$ and $c + a$ are in H.P., then the value of $2a + c$ is
    (a) 0 (b) 1 (c) 2 (d) 3

**14.** Let the roots of the equation $f(x) = x^5 - 10x^4 + ax^3 + bx^2 + cx - 32 = 0$ are positive. If $f(x)$ is divided by $x - 1$, then the remainder is

(a) $-1$ (b) 1 (c) 2 (d) $-2$

**15.** The total number of words formed by the letters of the word 'PARABOLA', if only two A's should come together, is

(a) 5040 (b) 3600 (c) 4320 (d) 2400

**16.** For all complex numbers $z_1$, $z_2$ satisfying $|z_1| = 15$ and $|z_2 - 4 - 3i| = 5$ then the minimum value of $|z_1 - z_2|$ is

(a) 0 (b) 5 (c) 10 (d) 15

**17.** $n$ arithmetic means are inserted between two sets of numbers $a, a^2$ & $b, b^2$ where $a, b \in R$. Suppose $m^{th}$ mean between these two sets of numbers is same, then $a + b$ equals

(a) $\dfrac{n+1-m}{n}$ (b) $\dfrac{n}{n+1-m}$

(c) $\dfrac{m+n-1}{m}$ (d) $\dfrac{m-n-1}{m}$

**18.** If $a, b, c$, are real and $x^3 - 3b^2x + 2c^3$ is divisible by $(x - a)$ and $(x - b)$, then $a + b + c$ is $(a \neq b)$

(a) $-2$ (b) $-1$ (c) 0 (d) 1

**19.** The value of

$$2\,{}^{10}C_0 + \frac{2^2}{2}\,{}^{10}C_1 + \frac{2^3}{3}\,{}^{10}C_2 + \frac{2^4}{4}\,{}^{10}C_3 + \ldots + \frac{2^{11}}{11}\,{}^{10}C_{10} = \frac{a^b - 1}{11},$$

then ${}^bC_a$ is,

(a) 55 (b) 165 (c) 330 (d) 11

**20.** Total number of ways of selecting four letters from the word 'ELLIPSE' is

(a) 18 (b) 21 (c) 42 (d) 44

**21.** The equation of the plane containing the lines $\dfrac{x-4}{1} = \dfrac{y+1}{1} = \dfrac{z-0}{-1}$ and $4x - y + 5z - 7 = 0 = 2x - 5y - z - 3$ is $ax + by + cz - 2 = 0$, then $a + b - c$ is equal to

(a) 0 (b) 1 (c) 2 (d) 4

**22.** The number of values of $\lambda$ ($\lambda \in$ R) for which $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is at right angle to each of the vectors $\vec{a} = \lambda\hat{i} + \hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \lambda\hat{k}$ and $\vec{c} = -2\hat{i} + \lambda\hat{j} + 3\hat{k}$

(a) 0 (b) 1 (c) 2 (d) 3

**23.** If pair of tangents drawn from any point $A(z_1)$ on the curve $|z - (3 + 4i)| = 2$ to the curve $|z - (3 + 4i)| = 1$, meeting at points $B(z_2)$ and $C(z_3)$, then distance between circumcentre and orthocentre of $\Delta ABC$ is

(a) 0 (b) $\dfrac{1}{2}$ (c) $\dfrac{1}{3}$ (d) $\dfrac{3}{4}$

**24.** If $(1 + x + 2x^2)^n = \sum_{r=0}^{2n} a_r x^r, n \in N$ then $a_0 + a_2 + a_4 + \ldots + a_{2n}$ is equal to

(a) 0 (b) $2^n(2^n - 1)$

(c) $2^{n-1}(2^n - 1)$ (d) $2^{n-1}(2^{n-1} - 1)$

**25.** The value of $\lim_{x\to 0}\left[\dfrac{\sin^2 x}{x\sin(\sin x)}\right]$ is equal to (where $[\cdot]$ represents greatest integer function)

(a) 0 (b) 1 (c) 2 (d) D.N.E

**26.** The minimum value of $4x^2 - 4x|\sin\theta| - \cos^2\theta$ is equal to $(x \in R)$

(a) $-2$ (b) $-1$ (c) $-1/2$ (d) 0

**27.** If $P_n = \cos^n\theta + \sin^n\theta$, then $P_6 - P_4 = KP_2$, where

(a) $K = 1$ (b) $K = -\sin^2\theta \cdot \cos^2\theta$

(c) $K=\sin^2\theta$ (d) $K = \cos^2\theta$

**28.** If '$\alpha$' is a root of the equation $4x^2 + 2x - 1 = 0$ and $f(x) = 4x^3 - 3x + 1$, then $2[f(\alpha) + \alpha]$ equal to

(a) 0 (b) $-1$ (c) 1 (d) 2

**29.** If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ then $\dfrac{dy}{dx}$ is

(a) $\dfrac{1}{(1+x)^2}$ (b) $\dfrac{-1}{(1+x)^2}$

(c) $\dfrac{1}{1+x}$ (d) $\dfrac{-1}{1+x}$

**30.** If ${}^nC_r$ denotes the number of combination of $n$ things taken $r$ at a time, then the expression ${}^nC_0 + \sum_{k=0}^{n-1} {}^{n+k}C_{k+1}$ equal

(a) ${}^{2n}C_{n-1}$ (b) ${}^{2n}C_{n+1}$ (c) ${}^{2n}C_n$ (d) ${}^nC_{n-1}$

## SOLUTIONS

**1.** **(b):** We have $\alpha + \beta = 2p$, $\alpha\beta = 2p^4$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= 4p^2 - 4p^4 = -4(p^4 - p^2)$

$$= -4\left[p^4 - p^2 + \frac{1}{4} - \frac{1}{4}\right] = 1 - 4\left(p^2 - \frac{1}{2}\right)^2 \le 1$$

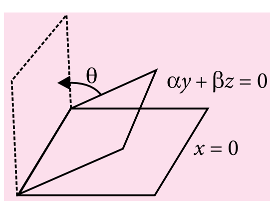
**2.** **(c):** Equation of a plane in new position, $\alpha y + \beta z + kx = 0$, ($k$ is any variable) DR's of normal to $\alpha y + \beta z + kx = 0$ is $k, \alpha, \beta$ and DR's of normal to $\alpha y + \beta z = 0$ is $0, \alpha, \beta$

$$\Rightarrow \quad \cos\theta = \frac{\alpha^2 + \beta^2}{\sqrt{k^2 + \alpha^2 + \beta^2}\sqrt{\alpha^2 + \beta^2}}$$

$$= \frac{\sqrt{\alpha^2 + \beta^2}}{\sqrt{k^2 + \alpha^2 + \beta^2}}$$

$$\Rightarrow \quad \sec^2\theta = \frac{k^2 + \alpha^2 + \beta^2}{\alpha^2 + \beta^2} = 1 + \frac{k^2}{\alpha^2 + \beta^2}$$

$$\Rightarrow \quad \tan^2\theta = \frac{k^2}{\alpha^2 + \beta^2} \Rightarrow k = \pm\sqrt{\alpha^2 + \beta^2}\tan\theta$$




**3.** **(a):** Term with greatest coefficient is

${}^{12}C_6 x^6 = T_7$

$$\left|\frac{T_7}{T_6}\right| > 1, \left|\frac{T_8}{T_7}\right| < 1 \Rightarrow \left|\frac{7}{6}x\right| > 1, \left|\frac{6}{7}x\right| < 1$$

$$\Rightarrow \quad |x| > \frac{6}{7}, |x| < \frac{7}{6} \quad \Rightarrow \ x \in \left(-\frac{7}{6}, -\frac{6}{7}\right) \cup \left(\frac{6}{7}, \frac{7}{6}\right)$$

**4.** **(d):** $$\lim_{x\to 0} \frac{-2\sin\left(\frac{\tan x + x}{2}\right)\sin\left(\frac{\tan x - x}{2}\right)}{x^4}$$

$$= \lim_{x\to 0} \frac{-2\sin\left(\frac{\tan x + x}{2}\right)}{\frac{\tan x + x}{2}} \frac{\sin\left(\frac{\tan x - x}{2}\right)}{\frac{\tan x - x}{2}} \cdot \frac{\tan^2 x - x^2}{4x^4}$$

$$= \frac{-2}{4}\lim_{x\to 0}\left(\frac{\tan x + x}{x}\right)\left(\frac{\tan x - x}{x^3}\right)$$

$$= -\frac{1}{2}\lim_{x\to 0}\left(\frac{\tan x}{x} + 1\right)\frac{\left(x + \frac{x^3}{3} \cdots - x\right)}{x^3} = -\frac{1}{3}$$

**5.** **(c):** $$I = \int_{-\ln 2}^{\ln 2} \frac{e^{\sin x}}{e^{\sin x} + e^{-\sin x}}dx$$

$$I = \int_{-\ln 2}^{\ln 2} \frac{e^{-\sin x}}{e^{-\sin x} + e^{\sin x}}dx$$

$$\left(\int_a^b f(x)dx = \int_a^b f(a + b - x)dx\right)$$

On adding,

$$2I = \int_{-\ln 2}^{\ln 2} \frac{e^{\sin x} + e^{-\sin x}}{e^{\sin x} + e^{-\sin x}}dx = \ln 2 - (-\ln 2)$$

$\Rightarrow \ 2I = 2\ln 2 \ \Rightarrow \ I = \ln 2$

**6.** **(b):** We have $[2\sin x] + [\cos x] = -3$

$\Rightarrow [2\sin x] = -2$ and $[\cos x] = -1$

$$\Rightarrow \ x \in \left(\frac{7\pi}{6}, \frac{11\pi}{6}\right) \text{and } x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$$

$$\Rightarrow \ x \in \left(\frac{7\pi}{6}, \frac{3\pi}{2}\right)$$

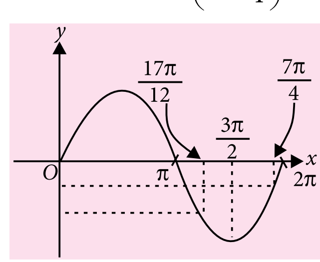
Now $g(x) = \sin x + \cos x = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$

Now when $\dfrac{7\pi}{6} < x < \dfrac{3\pi}{2}$

$$\frac{17\pi}{12} < x + \frac{\pi}{4} < \frac{7\pi}{4}$$

$$\Rightarrow -1 \le \sin\left(x + \frac{\pi}{4}\right) < -\frac{1}{\sqrt{2}}$$

$$\Rightarrow -\sqrt{2} \le \sqrt{2}\sin\left(x + \frac{\pi}{4}\right) < -1 \ \Rightarrow \ g(x) \in [-\sqrt{2}, -1)$$



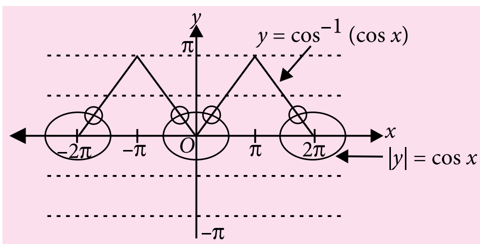
**7.** **(c) :** $gof\,(x) = g[f(x)] = g\,(\{x\} + \{-x\})$

$= \sin^{-1}(\{x\}+\{-x\}) = \sin^{-1}(0)$ or $\sin^{-1}(1)$

$= 0$ or $\dfrac{\pi}{2}$

So, number of elements in range of $gof\,(x) = 2$

**8.** **(d) :**

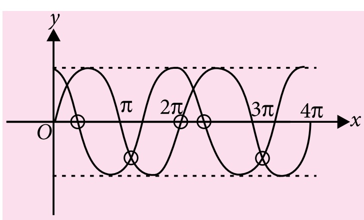



**9.** **(d) :** $fog\,(x) = f\,(g(x)) = [g(x) + 2] = [\cos x + 2]$

$= [\cos x] + 2$

Function is not continuous at $x = \dfrac{\pi}{2}$

**10.** **(a)**




No. of points of non-differentiability

in the interval $(0, 2\pi) = 2$

in the interval $(0, 4\pi) = 5$

in the interval $(0, 6\pi) = 8$

in the interval $(0, 2n\pi) = (3n - 1)$

**11.** **(d) :** $\arg(-1 + i)^{50} = 50 \arg(-1 + i)$

$= 50(\pi - \tan^{-1}(1)) = 50\left(\pi - \dfrac{\pi}{4}\right)$

$= 50 \times \dfrac{3\pi}{4} = \dfrac{75\pi}{2} = \dfrac{76\pi - \pi}{2} = 38\pi - \dfrac{\pi}{2}$

So the principle argument $= -\dfrac{\pi}{2}$

$\Rightarrow n = -1$

Now, $a = (1 + i)^{-n} = (1 + i)$

So equation becomes: $z^2 - (1 + i)\,z + b + 2i = 0$

Let suppose $\alpha$ is the real root of equation, then $\alpha^2 - (1 + i)\,\alpha + b + 2i = 0$

$\Rightarrow (\alpha^2 - \alpha + b) + i(2 - \alpha) = 0$

$\Rightarrow 2 - \alpha = 0$ and $\alpha^2 - \alpha + b = 0$

$\Rightarrow \alpha = 2$ and $4 - 2 + b = 0 \Rightarrow b = -2$

So the real root of the equation is $\alpha = 2$.

Now product of roots $= b + 2i$

$= -2 + 2i = 2(-1 + i)$

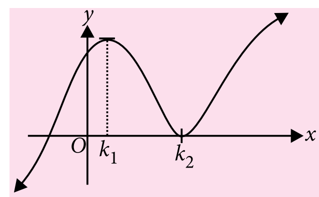
So the other root is $(-1 + i)$

**12.** **(c) :** $f(x) = x^3 + 3x^2 - 9x + c = 0$ ...(i)

$\Rightarrow f'(x) = 3x^2 + 6x - 9 = 0$ ...(ii)

$D = 36 - 4 \times 3 \times (-9) > 0$

$\Rightarrow f'(x) = 0$ has two roots, but (i) has two equal roots, so the graph of (i) will be as shown. If $k_1$ and $k_2$ are roots of $f'(x) = 0$, then $f(k_1) \cdot f(k_2) = 0$




$\Rightarrow f(-3) \cdot f(1) = 0$

$\Rightarrow (-27 + 27 + 27 + c)\,(1 + 3 - 9 + c) = 0$

$\Rightarrow (c + 27)\,(c - 5) = 0 \Rightarrow c = 5$ or $-27$

**13.** **(a) :** Let $a, b, c$ are of the type $a, ar, ar^2$

Now $a + b, b + c, c + a$ are in H.P.

$$\Rightarrow \frac{2}{b+c} = \frac{1}{a+b} + \frac{1}{c+a}$$

$$\Rightarrow \frac{2}{ar+ar^2} = \frac{1}{a+ar} + \frac{1}{ar^2+a}$$

$$\Rightarrow \frac{2}{r(1+r)} = \frac{1+r^2+1+r}{(1+r)(1+r^2)}$$

$\Rightarrow 2 + 2r^2 = r^3 + r^2 + 2r$

$\Rightarrow r^3 - r^2 + 2r - 2 = 0$

$\Rightarrow (r - 1)\,(r^2 + 2) = 0$

$\Rightarrow r \neq 1$ so, $r^2 + 2 = 0$

Now, $2a + c = 2a + ar^2 = a\,(2 + r^2) = 0$

**14.** **(a) :** Let the roots of equation are

$x_1, x_2, x_3, x_4, x_5$

Now, $x_1 + x_2 + x_3 + x_4 + x_5 = 10$ and

$x_1\,x_2\,x_3\,x_4\,x_5 = 32$

So, $\dfrac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = \text{A.M.} = 2$

$= (x_1 \cdot x_2 \cdot x_3 \cdot x_4 \cdot x_5)^{1/5} = \text{G.M.}$

$\Rightarrow$ A.M. = G.M.

$\Rightarrow x_1 = x_2 = x_3 = x_4 = x_5 = 2$

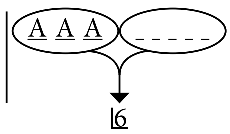
$\Rightarrow$ Given equation is $(x - 2)^5 = 0$

$\therefore$ Remainder $= f(1) = -1$

**15. (b):** Required no. of words = (total no. of words) – (No. of words when three A's come together) – (No. of words when no two A's come together) = $A - B - C$

Now $A = \frac{\lfloor 8}{\lfloor 3} = 6720$

$B = \lfloor 6 = 720$




Now,

$C = {}^6C_3 \times \lfloor 5$ | _P_R_B_O_L_can be arranged in $\lfloor 5$ ways

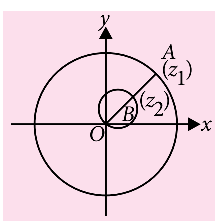
$= \frac{\lfloor 6}{\lfloor 3 \lfloor 3} \times \lfloor 5$ | We have 3 A's those can be placed at 6 places *i.e.*, ${}^6C_3$ ways. Remaining can be arranged in $\lfloor 5$ ways

$= 20 \times 120$

$= 2400$

Now, $A - B - C = 6720 - 720 - 2400 = 3600$

**16. (b):**




Min. value of $|z_1 - z_2| = AB = 15 - 2(5) = 5$

**17. (d):** $\frac{(n-m+1)a+ma^2}{n+1} = \frac{(n-m+1)b+mb^2}{n+1}$

$\Rightarrow (n - m + 1)(a - b) = m(b^2 - a^2)$

$\Rightarrow a+b = \frac{n-m+1}{-m} = \frac{m-n-1}{m}$

**18. (c):** Let $f(x) = x^3 - 3b^2x + 2c^3$

Now, $f(a) = a^3 - 3b^2a + 2c^3 = 0$ ...(i)

and $f(b) = b^3 - 3b^3 + 2c^3 = 0$

$\Rightarrow -2b^3 + 2c^3 = 0 \Rightarrow b = c$ ...(ii)

Putting in (i), we get

$a^3 - 3b^2a + 2b^3 = 0$

$\Rightarrow (a - b)(a^2 + ab - 2b^2) = 0$

$\Rightarrow a^2 + ab - 2b^2 = 0, a \neq b$

$\Rightarrow (a + 2b)(a - b) = 0$

$\Rightarrow a + 2b = 0, a \neq b$ ... (iii)

$\Rightarrow a + 2c = 0$ ...(iv)

Adding (iii) and (iv), we get

$2a + 2b + 2c = 0 \Rightarrow a + b + c = 0$

**19. (b):** $\sum_{r=0}^{10} {}^{10}C_r \cdot \frac{2^{r+1}}{r+1} = \sum_{r=0}^{10} \frac{\lfloor 10}{\lfloor r \lfloor 10-r} \frac{2^{r+1}}{r+1}$

$= \frac{1}{11} \sum_{r=0}^{10} \frac{\lfloor 11}{\lfloor r+1 \lfloor 11-(r+1)} \cdot 2^{r+1} = \frac{1}{11} \sum_{r=0}^{10} {}^{11}C_{r+1} \cdot 2^{r+1}$

$= \frac{1}{11}\left({}^{11}C_1 2^1 + {}^{11}C_2 2^2 + {}^{11}C_3 2^3 + \ldots + {}^{11}C_{11} 2^{11}\right)$

$= \frac{1}{11}\left[(1+2)^{11} - 1\right] = \frac{3^{11}-1}{11} = \frac{a^b - 1}{11}$

$\Rightarrow a = 3, b = 11$

Now, ${}^bC_a = {}^{11}C_3 = \frac{\lfloor 11}{\lfloor 3 \lfloor 8} = \frac{9 \cdot 10 \cdot 11}{2 \cdot 3} = 165$

**20. (a):** **Case - I :** All the letters are distinct = ${}^5C_4$ ways = 5

**Case - II :** 2 distinct and 2 alike

$= {}^2C_1 \times {}^4C_2 = 2 \times 6 = 12$

**Case - III :** 2 alike and 2 alike = 1

So total numbers of ways = 5 + 12 + 1 = 18

**21. (a):** Equation of plane passing through the intersection of planes $4x - y + 5z - 7 = 0$

$2x - 5y - z - 3 = 0$ is

$(4x - y + 5z - 7) + \lambda(2x - 5y - z - 3) = 0$

Plane passes through $(4, -1, 0)$, therefore $\lambda = -1$

Therefore, equation of plane is $x + 2y + 3z - 2 = 0$

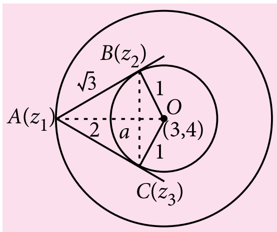
**22. (b):** $\vec{a}, \vec{b}$ and $\vec{c}$ must be coplanar vectors

$[\vec{a}\ \vec{b}\ \vec{c}] = 0$

$$\begin{vmatrix} \lambda & 1 & 3 \\ 2 & 1 & -\lambda \\ -2 & \lambda & 3 \end{vmatrix} = 0$$

$\Rightarrow \lambda^3 + 11\lambda = 0 \Rightarrow \lambda = 0$

**23. (a) :**




$OA = 2$, $OB = 1$, $AB = \sqrt{3}$

$\Delta ABC$ is equilateral triangle.

Therefore, distance between orthocentre and circumcentre is zero.

**24. (c) :** Put $x = 1$, $a_0 + a_1 + a_2 + ... + a_{2n} = 4^n$

put $x = -1$, $a_0 - a_1 + a_2 - ...\ a_{2n} = 2^n$

$\therefore\ 2(a_0 + a_2 + ... + a_{2n}) = 4^n - 2^n$

$\Rightarrow a_0 + a_2 + ... + a_{2n} = 2^{n-1}(2^n - 1)$

**25. (a) :** Let $f(x) = \dfrac{x}{\sin x}$

$$f'(x) = \frac{\sin x - x\cos x}{(\sin x)^2} = \frac{\cos x(\tan x - x)}{(\sin x)^2}$$

When $x > 0$, then $\tan x > x$ and $\cos x > 0$, therefore, $f'(x) > 0$, $f(x)$ is increasing.

Since $f(x) > f(\sin x)$ for $x > \sin x$

$$\therefore\ \frac{x}{\sin x} > \frac{\sin x}{\sin(\sin x)} \quad \Rightarrow \quad 1 > \frac{(\sin x)^2}{x\sin(\sin x)}$$

Similarly, $x < 0$, $\dfrac{(\sin x)^2}{x\sin(\sin x)} < 1$

**26. (b) :** Minimum value $= \dfrac{-16\cos^2\theta - 16\sin^2\theta}{4\times 4}$

$= -(\cos^2\theta + \sin^2\theta) = -1$

$$\left\{\text{Using } ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{D}{4a} \geq \frac{-D}{4a} \text{ for } a > 0\right\}$$

**27. (b) :** $P_6 - P_4 = KP_2$

$\Rightarrow (\sin^6\theta + \cos^6\theta) - (\sin^4\theta + \cos^4\theta)$

$= K(\sin^2\theta + \cos^2\theta)$

$\Rightarrow K = (1 - 3\sin^2\theta\cdot\cos^2\theta) - (1 - 2\sin^2\theta\cdot\cos^2\theta)$

$\Rightarrow K = -\sin^2\theta\cdot\cos^2\theta$

**28. (c) :** Given that, $4\alpha^2 = 1 - 2\alpha$

So, $f(\alpha) = 4\alpha^3 - 3\alpha + 1$

$= \alpha(1 - 2\alpha) - 3\alpha + 1 = -2\alpha^2 - 2\alpha + 1$

$$= -\left(\frac{1-2\alpha}{2}\right) - 2\alpha + 1 = -\alpha + \frac{1}{2}$$

$$\text{Now, } 2[f(\alpha) + \alpha] = 2\left(-\alpha + \frac{1}{2} + \alpha\right) = 1$$

**29. (b) :** Here, $x\sqrt{1+y} = -y\sqrt{1+x}$

$\Rightarrow x^2(1 + y) = y^2(1 + x)$

$\Rightarrow x^2 - y^2 = y^2x - x^2y$

$\Rightarrow x + y = -xy \quad \Rightarrow \quad y = \dfrac{-x}{1+x} = \dfrac{1}{1+x} - 1$

So, $\dfrac{dy}{dx} = \dfrac{-1}{(1+x)^2}$

**30. (c) :** ${}^nC_0 + \sum_{k=0}^{n-1} {}^{n+k}C_{k+1}$

$= \underbrace{{}^nC_0 + {}^nC_1} + {}^{n+1}C_2 + {}^{n+2}C_3 + ... + {}^{2n-1}C_n$

$= \underbrace{{}^{n+1}C_1 + {}^{n+1}C_2} + {}^{n+2}C_3 + ... + {}^{2n-1}C_n$

$= \underbrace{{}^{n+2}C_2 + {}^{n+2}C_3} + ... + {}^{2n-1}C_n$

$.... = {}^{2n}C_n$

# JEE WORK OUTS

## PAPER-I

**1.** Let $f : R \to R$ be a differentiable function satisfying $f(x) = f(y)\, f(x - y)\ \forall\ x, y \in R$ and $f'(0) = \int_0^4 \{2x\}dx$, where $\{\cdot\}$ denotes the fractional part function and $f'(-3) = \alpha e^{\beta}$. Then, $|\alpha + \beta|$ is equal to ________

**2.** If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors, then find the maximum value of $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$.

**3.** If $\int \frac{\ln x}{x^3} dx = -\frac{\ln x}{ax^2} - \frac{1}{bx^2} + c$, then the value of $a + b$ must be

**4.** $TP$ and $TQ$ are any two tangents to a parabola and the tangent at a third point $R$ cuts them in $P'$ and $Q'$, then the value of $\frac{TP'}{TP} + \frac{TQ'}{TQ}$ must be

**5.** If $\alpha$, $\beta$, $\gamma$, $\delta$ are the solution of the equation $\tan\left(\theta + \frac{\pi}{4}\right) = 3\tan 3\theta$, no two of which have equal tangents, then the value of $\tan\alpha + \tan\beta + \tan\gamma + \tan\delta$ is equal to

**6.** The number of solutions for the equation $\log_4(2x^2 + x + 1) - \log_2(2x - 1) = 1$, is

**7.** Find the value of
$$4\left(\cos\frac{2\pi}{15} + \cos\frac{4\pi}{15} - \cos\frac{7\pi}{15} - \cos\frac{\pi}{15}\right).$$

**8.** If $(y^2 - 5y + 3)(x^2 + x + 1) < 2x$ for all $x \in R$, then number of integral values of $y$ will be

**9.** If $a, b, c$ are real numbers and $z$ is a complex number such that $a^2 + b^2 + c^2 = 1$ and $b + ic = (1 + a)z$, then $\frac{1+iz}{1-iz}$ equals

(a) $\frac{b-ic}{1-ia}$ (b) $\frac{a+ib}{1+c}$ (c) $\frac{1-c}{a-ib}$ (d) $\frac{1+a}{b+ic}$

**10.** If the quadratic polynomial $P(x) = (p - 3)x^2 - 2px + 3p - 6$ ranges from $[0, \infty)$ for every $x \in R$, then the value of $p$ can be

(a) $\frac{3}{2}$ (b) 4 (c) 6 (d) 7

**11.** All non-zero complex numbers on the complex plane satisfying $\left(z + \frac{1}{z}\right) \in R$ can lie on

(a) $y = 0$
(b) $x = 0$
(c) the line $y = x$
(d) a unit circle with centre at the origin

**12.** Domain of $f(x) = \sin^{-1}[2 - 4x^2]$ is ($[\cdot]$ denotes the greatest integer function)

(a) $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right] - \{0\}$ (b) $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right]$

(c) $\left[-\frac{\sqrt{3}}{2}, 0\right)$ (d) $\left[-\frac{\sqrt{3}}{2}, 8\right]$

**13.** $\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j} 1$ is equal to

(a) $\frac{n(n+1)(n+2)}{6}$ (b) $\sum_{n=1}^{n} n^2$

(c) $\frac{n(n+1)(2n+1)}{6}$ (d) ${}^{n+2}C_3$

**14.** The line $3x + 6y = k$ intersect the curve $2x^2 + 2xy + 3y^2 = 1$ at points $A$ and $B$. The circle on $AB$ as diameter passes through the origin. The possible value(s) of $k$ is (are)

(a) 3 (b) 4 (c) −4 (d) −3

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**15.** Let $P(x) = x^2 + bx + c$, where $b$ and $c$ are integer. If $P(x)$ is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$, then

(a) $P(x) = 0$ has imaginary roots
(b) $P(x) = 0$ has roots of opposite sign
(c) $P(1) = 4$
(d) $P(1) = 6$

**16.** $$[({}^nC_0 + {}^nC_3 + ...) - \frac{1}{2}({}^nC_1 + {}^nC_2 + {}^nC_4 + {}^nC_5 + ...)]^2 + \frac{3}{4}({}^nC_1 - {}^nC_2 + {}^nC_4 - {}^nC_5 + ...)^2 =$$

(a) 3 (b) 4 (c) 2 (d) 1

**17.** For every pair of continuous functions $f, g$: $[0, 1] \to R$, such that $\max\{f(x) : x \in [0, 1]\} = \max\{g(x) : x \in [0, 1]\}$, the correct statement(s) is (are):

(a) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0, 1]$
(b) $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0, 1]$
(c) $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some $c \in [0, 1]$
(d) $(f(c))^2 = (g(c))^2$ for some $c \in [0, 1]$

**18.** Let $\overrightarrow{PR} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\overrightarrow{SQ} = \hat{i} - 3\hat{j} - 4\hat{k}$ determine diagonals of a parallelogram $PQRS$ and $\overrightarrow{PT} = \hat{i} + 2\hat{j} + 3\hat{k}$ be another vector. Then the volume of the parallelopiped determined by the vectors $\overrightarrow{PT}$, $\overrightarrow{PQ}$ and $\overrightarrow{PS}$ is

(a) 5 (b) 20 (c) 10 (d) 30

**19.** Match the following :

| Column I | Column II |
|---|---|
| (a) Consider the differential equation, $y = x\frac{dy}{dx} + \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ Its degree is | (p) 1 |
| (b) The differential equation representing the family of curve $y^2 = 2c(x + \sqrt{c})$, where $c$ is a parameter is of degree | (q) 4 |
| (c) Consider differential equation, $1 + \left(\frac{dy}{dx}\right) + \frac{1}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{1}{3!}\left(\frac{dy}{dx}\right)^3 + ...\infty = y$ then its degree is | (r) 2 |
| (d) Consider the differential equation, $\frac{dy}{dx} = \left(1 + \left(\frac{d^2y}{dx^2}\right)^2\right)^{2/3}$. It's degree is | (s) 3 |

**20.** Consider the system of equations

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

Now, let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $\Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$

$\Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$ and $\Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$

According to Crammer's rule,

$$x = \frac{\Delta_1}{\Delta},\ y = \frac{\Delta_2}{\Delta},\ z = \frac{\Delta_3}{\Delta}$$

Match the Following :

| Column I | Column II |
|---|---|
| (a) If $\Delta \neq 0$ and $\Delta_1 = \Delta_2 = \Delta_3 = 0$, then the solutions are | (p) Unique |
| (b) If $\Delta_1 = \Delta_2 = \Delta_3 = \Delta = 0$, then the solutions are | (q) Inconsistent |
| (c) If $\Delta \neq 0$ and atleast one of $\Delta_1$, $\Delta_2$, $\Delta_3$ is non-zero, then the solutions are | (r) Non-Trivial |
| (d) If atleast one of $\Delta_1$, $\Delta_2$, $\Delta_3$ is non-zero, and $\Delta = 0$, then the solutions are | (s) Infinitely many |

## PAPER - II

1. Let $f(x, y)$ be a periodic function satisfying the condition $f(x, y) = f((2x + 2y), (2y - 2x))\ \forall\ x, y \in R$. Now, define a function $g$ by $g(x) = f(2^x, 0)$. If period of $g(x)$ function is $\alpha$, then value of $|\alpha - 7|$ is

2. For a point $P$ in the plane, let $d_1(P)$ and $d_2(P)$ be the distances of the point $P$ from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region $R$ consisting of all points $P$ lying in the first quadrant of the plane and satisfying $2 \le d_1(P) + d_2(P) \le 4$, is

3. The circle $x^2 + y^2 - 4x - 4y + 4 = 0$ is inscribed in a triangle which has two of its sides along the coordinate axes. The locus of the circumcentre of the triangle is $x + y - xy + k\sqrt{x^2 + y^2} = 0$, find $k$.

4. If $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$ for all $x \in R$, then find the value of $f(1)$.

5. Find $\lim\limits_{x \to -\infty} \left| \dfrac{(3x^4 + 2x^2)\sin\frac{1}{x} + |x|^3 + 5}{|x|^3 + |x|^2 + |x| + 1} \right|$

6. If $y(t)$ is a solution of $(1+t)\dfrac{dy}{dt} - ty = 1$ and $y(0) = -1$, then $y(1) + \dfrac{3}{2}$ is equal to

7. Suppose that $f$ is differentiable for all $x$ and that $f'(x) \le 2$ for all $x$. If $f(1) = 2$ and $f(4) = 8$, then $f(2)$ has the value equal to _______.

8. Two whole numbers are randomly selected and multiplied. If the probability that the units place in their product is "Even" is $p$ and the probability that the units place in their product is "odd" is $q$, then $\left(\dfrac{p}{q}\right) + 1$ is equal to

9. The point of intersection of the plane $\vec{r} \cdot (3\hat{i} - 5\hat{j} + 2\hat{k}) = 6$ with the straight line passing through the origin and perpendicular to the plane $2x - y - z = 4$ is
   (a) $(1, -1, -1)$ (b) $(-1, -1, 2)$
   (c) $(4, 2, 2)$ (d) $\left(\dfrac{4}{3}, \dfrac{-2}{3}, \dfrac{-2}{3}\right)$

10. If the median through $A$ of a $\Delta ABC$ having vertices $A \equiv (2, 3, 5)$, $B \equiv (-1, 3, 2)$ and $C \equiv (\lambda, 5, \mu)$ is equally inclined to the axes, then
    (a) $\lambda = 7$ (b) $\mu = 10$
    (c) $\lambda = 10$ (d) $\mu = 7$

11. The integral of $\dfrac{1}{\sin^2 x + \tan^2 x}$ must be
    (a) $-\dfrac{1}{2}\left[\tan x + \dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{1}{\sqrt{2}}\tan x\right)\right] + c$
    (b) $-\dfrac{1}{2}\left[\cot x + \dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{1}{\sqrt{2}}\tan x\right)\right] + c$
    (c) $-\left[\cot x - \dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{1}{\sqrt{2}}\tan x\right)\right] + c$
    (d) $-\dfrac{1}{2}\left[\cot x - \dfrac{1}{\sqrt{2}}\cot^{-1}\left(\dfrac{1}{\sqrt{2}}\tan x\right)\right] + c$

12. $I = \displaystyle\int \sqrt{\frac{\cos^3 x}{\sin^{11} x}}\, dx$ is equal to
    (a) $-\dfrac{2}{5}\cot^{3/2} x - \dfrac{2}{9}\cot^{9/2} x + c$
    (b) $-\dfrac{2}{5}\cot^{5/2} x - \dfrac{2}{9}\cot^{9/2} x + c$
    (c) $-\dfrac{2}{5}\cot^{5/2} x - \dfrac{2}{9}\cot^{3/2} x + c$
    (d) none of these

13. The line $y = x + 5$ touches
    (a) the parabola $y^2 = 20x$
    (b) the ellipse $9x^2 + 16y^2 = 144$
    (c) the hyperbola $\dfrac{x^2}{29} - \dfrac{y^2}{4} = 1$
    (d) the circle $x^2 + y^2 = 25$

14. How many numbers between 5,000 and 10,000 can be formed using the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 each digit appearing not more than once in each number?
    (a) 1680 (b) $5 \times {}^8P_8$
    (c) $30 \times {}^8C_3$ (d) $5! \times {}^8C_3$

15. Triplets $(p, q, r)$ is chosen from a set {1, 2, 3, 4, 5, 6, 7, 8, 9} such that $x < y \le z$. There could be _______ such triplets.
    (a) 143 (b) 200 (c) 180 (d) 120

16. If in triangle $ABC$, $CD$ is the angle bisector of the angle $ACB$, then $CD$ is equal to
    (a) $\dfrac{a+b}{2ab}\cos\dfrac{C}{2}$ (b) $\dfrac{a+b}{ab}\cos\dfrac{C}{2}$
    (c) $\dfrac{2ab}{a+b}\cos\dfrac{C}{2}$ (d) $\dfrac{b\sin A}{\sin\left(B + \frac{C}{2}\right)}$

**Paragraph for Questions Nos. 17 and 18**

Consider a hyperbola whose centre is at origin. A line $x + y = 2$ touches this hyperbola at $P(1, 1)$ and intersect the asymptotes at $A$ and $B$ such that $AB = 6\sqrt{2}$ units.

**17.** Equation of asymptotes are

(a) $2x^2 + 5xy + 2y^2 = 0$ (b) $2x^2 - 5xy - 2y^2 = 0$

(c) $2x^2 - 5xy + 2y^2 = 0$ (d) $2x^2 + 5xy - 2y^2 = 0$

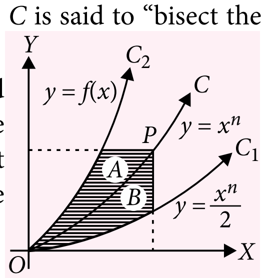
**18.** Equation of tangent to the hyperbola at $\left(-1, \frac{7}{2}\right)$ is

(a) $3x + 2y = 2$ (b) $3x + 4y = 11$

(c) $3x + 2y = 4$ (d) $3x + 2y = 7$

**Paragraph for Q. No. 19 to 20**

Let $C_1$ and $C_2$ be two curves passing through the origin as shown in the figure. A curve $C$ is said to "bisect the area" between $C_1$ and $C_2$, if for each point $P$ of $C$, the two shaded regions $A$ and $B$ shown in the figure have equal areas. Let the equation of the curve $C_1$ be $y = \frac{x^n}{2}$ and that of $C$ be $y = x^n$.




**19.** Area of the total shaded region ($A + B$), if $x$-coordinate of $P$ is $x$ and $n = 2$ is

(a) $\frac{x^2}{2}$ (b) $\frac{x^3}{2}$

(c) $\frac{x^3}{3}$ (d) $\frac{x^3}{6}$

**20.** Equation of the curve $C_2$ for $n = 2$ is $y =$

(a) $\frac{3x^2}{4}$ (b) $\frac{4x^2}{3}$

(c) $\frac{9x^2}{16}$ (d) $\frac{16x^2}{9}$

## ANSWER KEYS

### PAPER-I

**1.** (4) **2.** (9) **3.** (6) **4.** (1) **5.** (0)
**6.** (1) **7.** (2) **8.** (2) **9.** (b), (c)
**10.** (c) **11.** (a), (d) **12.** (a)
**13.** (a), (d) **14.** (a), (d)
**15.** (a), (c) **16.** (d) **17.** (a), (d) **18.** (c)
**19.** (a) → (r), (b) → (s), (c) → (p), (d) → (q)

### PAPER-II

**1.** (5) **2.** (6) **3.** (1) **4.** (4) **5.** (2)
**6.** (1) **7.** (4) **8.** (4) **9.** (d)
**10.** (a), (b) **11.** (b), (d) **12.** (b)
**13.** (a), (b), (c) **14.** (a), (c) **15.** (d)
**16.** (c), (d) **17.** (a) **18.** (c) **19.** (c)
**20.** (d)

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# PRACTICE PAPER 2016
# JEE MAIN

1. If $(f(x))^2 f\left(\frac{1-x}{1+x}\right) = x^3, x \neq -1, 1$ and $f(x) \neq 0$, then $f(x)$ is
(a) $x^2\left(\frac{1-x}{1+x}\right)$ (b) $x^2\left(\frac{1+x}{1-x}\right)$
(c) $x^2\left(\frac{1-2x}{1+x}\right)$ (d) None of these

2. If $(\alpha + \beta i)^{11} = x + iy$, where $\alpha, \beta, x, y \in R$, then $Re((\beta + \alpha i)^{11})$ equals
(a) $y$ (b) $-y$ (c) $x$ (d) $-x$

3. Suppose $a, b, c \in R$ and $b \neq c$. If $\alpha, \beta$ are roots of $x^2 + ax + b = 0$ and $\gamma, \delta$ are roots of $x^2 + ax + c = 0$, then the equation whose roots are $\frac{(\alpha-\gamma)(\alpha-\delta)}{(\beta-\gamma)(\beta-\delta)}$ and 2 is
(a) $x^2 + x + 1 = 0$ (b) $x^2 - x + 1 = 0$
(c) $x^2 - 3x + 2 = 0$ (d) $x^2 - 3x - 2 = 0$

4. Let $\Delta(x) = \begin{vmatrix} \sin x & \cos x & \sin 2x + \cos 2x \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{vmatrix}$ then $\Delta'(x)$ vanishes at least once in
(a) $(0, \pi/2)$ (b) $(\pi/2, \pi)$
(c) $(0, \pi/4)$ (d) $(-\pi/2, 0)$

5. Let $A$ be a $3 \times 3$ matrix such that $\det(A) = -2$. Then $\det(-2A^{-1}) + 2\det(A)$ is equal to
(a) $-2$ (b) $-4$
(c) 7 (d) None of these

6. The number of ways in which we can arrange the digits 1, 2, 3, ……., 9 such that the product of five digits at any of the five consecutive positions is divisible by 7 is
(a) 7! (b) ${}^9P_7$ (c) 8! (d) 5(7!)

7. Coefficient of $x^{17}$ in the polynomial
$P(x) = \prod_{r=0}^{17} (x + {}^{35}C_r)$ is
(a) $2^{34}$ (b) ${}^{36}C_{17}$
(c) $2^{35} - {}^{36}C_{17}$ (d) 0

8. If $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + ....... = a$, then value of
$\frac{1}{1\cdot3} + \frac{1}{5\cdot7} + \frac{1}{9\cdot11} + ........$ is
(a) $\frac{a}{2}$ (b) $\frac{a}{3}$ (c) $\frac{a}{4}$ (d) $a$

9. Let $a_n$ be the $n^{\text{th}}$ term of an A.P. with common difference $d$. If $\sum_{r=1}^{100} a_{2r} = \alpha$ and $\sum_{r=1}^{100} a_{2r-1} = \beta$, then $(\alpha - \beta) - 100d =$
(a) 0 (b) 50
(c) 100 (d) None of these

10. If $\lim_{x \to 0} \frac{(\cos x)^{1/2} - (\cos x)^{1/3}}{\sin^2 x} = a$, then the value of $12a + 2$ is
(a) 1 (b) $-1$
(c) 2 (d) None of these

11. Let $f$ be a differentiable function such that $8f(x) + 6f\left(\frac{1}{x}\right) - x = 5\ (x \neq 0)$ and $y = x^2 f(x)$, then $\frac{dy}{dx}$ at $x = 1$ is
(a) $\frac{15}{14}$ (b) $\frac{17}{14}$
(c) $\frac{19}{14}$ (d) $-\frac{17}{14}$

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12. If $f(x) = x^{2/3}$, then
(a) (0, 0) is a point of maximum
(b) (0, 0) is not a point of minimum
(c) (0, 0) is a critical point
(d) There is no critical point

13. The difference between the greatest and least values of the function

$f(x) = \cos x + \left(\frac{1}{2}\right)\cos 2x - \left(\frac{1}{3}\right)\cos 3x$ is

(a) $\frac{3}{8}$ (b) $\frac{2}{3}$ (c) $\frac{8}{7}$ (d) $\frac{9}{4}$

14. $\int_0^{\pi/4} \ln(1+\tan^2\theta+2\tan\theta)\,d\theta = \frac{\pi\ln a}{b}$ then the value of $2a + 3b$ is
(a) 15 (b) 16
(c) 17 (d) None of these

15. The solution of $\frac{dy}{dx} = \frac{1}{2x-y^2}$ is given by

(a) $y = Ce^{-2x} + \frac{1}{4}x^2 + \frac{1}{2}x + \frac{1}{4}$

(b) $x = Ce^{-y} + \frac{1}{4}y^2 + \frac{1}{4}y + \frac{1}{2}$

(c) $x = Ce^{y} + \frac{1}{4}y^2 + y + \frac{1}{2}$

(d) $x = Ce^{2y} + \frac{1}{2}y^2 + \frac{1}{2}y + \frac{1}{4}$

16. The incentre of triangle with vertices $(-\sqrt{3}, -1)$ (0, 0) and (0, −2) is

(a) $\left(-\sqrt{3}, -1\right)$ (b) $\left(-\frac{1}{\sqrt{3}}, -1\right)$

(c) $\left(-\frac{2}{\sqrt{3}}, -1\right)$ (d) None of these

17. Two tangents are drawn from the origin to a circle with centre at (2, – 1). If the equation of one of the tangents is $3x + y = 0$, the equation of the other tangent is
(a) $3x - y = 0$ (b) $x + 3y = 0$
(c) $x - 3y = 0$ (d) $x + 2y = 0$

18. If $PQ$ is a double ordinate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that $OPQ$ is an equilateral triangle, $O$ being the centre of the hyperbola, then eccentricity '$e$' of the hyperbola satisfies

(a) $e = \frac{2}{\sqrt{3}}$ (b) $e = \frac{\sqrt{3}}{2}$

(c) $e > \frac{2}{\sqrt{3}}$ (d) $1 < e < \frac{2}{\sqrt{3}}$

19. The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{1}{8}$ (d) $\frac{2}{3}$

20. The length of the perpendicular drawn from the point (3, – 1, 11) to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is

(a) $\sqrt{29}$ units (b) $\sqrt{33}$ units
(c) $\sqrt{53}$ units (d) $\sqrt{66}$ units

21. The value of $k$ for which the line $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies in the plane $2x - 4y + z = 7$ is
(a) 7 (b) 6
(c) no real value (d) – 7

22. If a variable takes the values 0, 1, 2, ……., $n$ with frequencies proportional to the binomial coefficients ${}^nC_0$, ${}^nC_1$, ………, ${}^nC_n$ then the mean of the distribution is

(a) $\frac{n(n+1)}{4}$ (b) $\frac{n}{2}$

(c) $\frac{n(n-1)}{2}$ (d) $\frac{n(n+1)}{2}$

23. Four natural numbers are selected at random and are multiplied. The probability that the product is divisible by 5 or 10 is

(a) $\frac{49}{625}$ (b) $\frac{369}{625}$ (c) $\frac{64}{625}$ (d) $\frac{256}{625}$

24. Each of two persons $A$ and $B$ toss three fair coins. The probability that both get the same number of heads is

(a) $\frac{3}{8}$ (b) $\frac{1}{9}$

(c) $\frac{5}{16}$ (d) None of these

25. If $\cos A = \frac{3}{4}$, then the value of

$16\cos^2\left(\frac{A}{2}\right) - 32\sin\left(\frac{A}{2}\right)\sin\left(\frac{5A}{2}\right)$ is

(a) – 4 (b) – 3 (c) 3 (d) 4

26. If $0 \le a,\ b \le 3$ and the equation $x^2 + 4 + 3\cos(ax + b) = 2x$ has at least one solution, then the locus of the point $(a, b)$ is
(a) $x + y = 2\pi$ (b) $x + y = \pi$
(c) $x^2 + y^2 = \pi^2$ (d) $y^2 = 2x$

27. A tower is standing at the centre of an elliptic field. If Aditya observes that the angle of elevation of the top of the tower at an extremity of the major axis of the field is $\alpha$, at its focus is $\beta$ and an extremity of the minor axis is $\gamma$, then
(a) $\cot^2\alpha = \cot^2\beta - \cot^2\gamma$
(b) $\cot^2\beta = \cot^2\gamma - \cot^2\alpha$
(c) $\cot^2\gamma = \cot^2\alpha - \cot^2\beta$
(d) None of these

28. **Statement I :** $\sim(p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$
**Statement II :** $\sim(p \leftrightarrow \sim q)$ is a tautology.
(a) Statement I is true, Statement II is true ; Statement II is a correct explanation for Statement I.
(b) Statement I is true, Statement II is true ; Statement II is not a correct explanation for Statement I.
(c) Statement I is true, Statement II is false.
(d) Statement I is false, Statement II is true.

29. **Statement I :** The point $A(1, 0, 7)$ is the mirror image of the point $B(1, 6, 3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.
**Statement II :** The line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ bisects the line segment joining $A(1, 0, 7)$ and $B(1, 6, 3)$.
(a) Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I.
(b) Statement I is true; Statement II is true; Statement II is not a correct explanation for Statement I.
(c) Statement I is true, Statement II is false.
(d) Statement I is false, Statement II is true.

30. Suppose $a, b, c \in R$
**Statement I :** If $z = a + (b + ic)^{2017} + (b - ic)^{2017}$ then $z$ is real.
**Statement II :** If $z = \bar{z}$, then $z$ is real.
(a) Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I.
(b) Statement I is true; Statement II is true; Statement II is not a correct explanation for Statement I.
(c) Statement I is true, Statement II is false.
(d) Statement I is false, Statement II is true.

## SOLUTIONS

1. **(b) :** Replacing $x$ by $\frac{1-x}{1+x}$ in the equation

$$(f(x))^2 f\left(\frac{1-x}{1+x}\right) = x^3 \quad \text{...(i)}$$

We have $$\left(f\left(\frac{1-x}{1+x}\right)\right)^2 f(x) = \left(\frac{1-x}{1+x}\right)^3 \quad \text{...(ii)}$$

Solving (i) and (ii), we have

$$\left(\frac{x^3}{(f(x))^2}\right)^2 f(x) = \left(\frac{1-x}{1+x}\right)^3$$

$$\Rightarrow \frac{(f(x))^3}{x^6} = \left(\frac{1+x}{1-x}\right)^3 \Rightarrow f(x) = x^2\left(\frac{1+x}{1-x}\right)$$

2. **(b) :** $\overline{(\alpha + \beta i)^{11}} = \overline{x + iy}$
$\Rightarrow (\alpha - \beta i)^{11} = x - iy$
$\Rightarrow [(-i)(\beta + i\alpha)]^{11} = -i(y + ix)$
$\Rightarrow (-i)^{11}(\beta + i\alpha)^{11} = -i(y + ix)$
We get $(\beta + i\alpha)^{11} = -(y + ix) = -y - ix$
Hence $\text{Re}((\beta + i\alpha)^{11}) = -y$

3. **(c) :** $x^2 + ax + c = (x - \gamma)(x - \delta)$

Thus, $$\frac{(\alpha-\gamma)(\alpha-\delta)}{(\beta-\gamma)(\beta-\delta)} = \frac{\alpha^2 + a\alpha + c}{\beta^2 + a\beta + c}$$
$$= \frac{-b+c}{-b+c} = 1 \ [\because \alpha, \beta \text{ are roots of } x^2 + ax + b = 0]$$

Hence for the required equation sum of root $= 1 + 2 = 3$ and product of root is $1 \times 2 = 2$.
So the equation is $x^2 - 3x + 2 = 0$

4. **(a) :** The function $\Delta(x)$ is continuous on $[0, \pi/2]$ and differentiable on $(0, \pi/2)$. Also $\Delta(0) = 0$ and $\Delta\left(\frac{\pi}{2}\right) = 0$. Thus, by the Rolle's theorem there exists at least one $c \in (0, \pi/2)$ such that $\Delta'(c) = 0$.

5. **(d) :** As $A^{-1}$ is a $3 \times 3$ matrix,
$\det(-2A^{-1}) = (-2)^3 \det(A^{-1}) = (-2)^3 (\det(A))^{-1}$
$$= (-8)\left(-\frac{1}{2}\right) = 4$$
Hence $\det(-2A^{-1}) + 2\det A = 4 + 2 \times (-2) = 0$

6. **(c) :** Let an arrangement of 9 digit number be $x_1\, x_2\, x_3\, x_4\, x_5\, x_6\, x_7\, x_8\, x_9$. Note that we require product of 5 consecutive digits. This is possible in the following ways :
$(x_1, x_2, x_3, x_4, x_5)$; $(x_2, x_3, x_4, x_5, x_6)$; .... ..; $(x_5, x_6, x_7, x_8, x_9)$

if the product of 5 consecutive digits is divisible by 7.

This is possible if $x_5$ is 7.

Therefore, we can arrange the 9 digits in desired number of ways in 8! ways.

**7. (a) :** $P(x)$ is a polynomial of degree 18, and coefficient of $x^{17}$ is

$$N = {}^{35}C_0 + {}^{35}C_1 + {}^{35}C_2 + \dots\dots\dots + {}^{35}C_{17} \quad \dots(1)$$

Using ${}^nC_r = {}^nC_{n-r}$, we can write $N$ as

$$N = {}^{35}C_{35} + {}^{35}C_{34} + \dots\dots\dots + {}^{35}C_{18} \quad \dots(2)$$

Adding (1) and (2), we get

$$2N = {}^{35}C_0 + {}^{35}C_1 + \dots\dots\dots + {}^{35}C_{35} = 2^{35}$$

$$\Rightarrow \; N = 2^{34}$$

**8. (a) :** We have

$$a = \left(1-\frac{1}{3}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)+\left(\frac{1}{9}-\frac{1}{11}\right)+\dots\dots$$

$$= \frac{2}{1\cdot3}+\frac{2}{5\cdot7}+\frac{2}{9\cdot11}+\dots\dots$$

$$\Rightarrow \; \frac{1}{1\cdot3}+\frac{1}{5\cdot7}+\frac{1}{9\cdot11}+\dots\dots\dots = \frac{a}{2}$$

**9. (a) :** Let $d$ be the common difference of the A.P., then

$$\alpha - \beta = \sum_{r=1}^{100}(a_{2r}-a_{2r-1}) = \sum_{r=1}^{100} d = 100d$$

$$\Rightarrow \; (\alpha - \beta) - 100d = 0$$

**10. (a) :** $a = \lim\limits_{x\to0}\dfrac{(\cos x)^{1/2}-(\cos x)^{1/3}}{\sin^2 x}\left(\dfrac{0}{0}\text{form}\right)$

$$\Rightarrow \; a = \lim_{x\to0}\frac{-\frac{1}{2}(\cos x)^{-1/2}\sin x+\frac{1}{3}(\cos x)^{-2/3}\sin x}{2\sin x\cos x}$$

$$\Rightarrow \; a = \lim_{x\to0}\frac{-\frac{1}{2}(\cos x)^{-1/2}+\frac{1}{3}(\cos x)^{-2/3}}{2\cos x}$$

$$\Rightarrow \; a = -\frac{1}{2}\left[\frac{1}{2}-\frac{1}{3}\right] = -\frac{1}{12}$$

$$\therefore \; 12a + 2 = -1 + 2 = 1$$

**11. (c) :** Differentiating the given expression, we get

$$8f'(x) + 6f'\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right) - 1 = 0$$

$$\Rightarrow \; 8f'(1) + 6f'(1)(-1) = 1 \Rightarrow f'(1) = \frac{1}{2}$$

Also $\dfrac{dy}{dx} = 2xf(x) + x^2 f'(x)$

So, $\left.\dfrac{dy}{dx}\right|_{x=1} = 2f(1) + f'(1)$

Putting $x = 1$ in the given equation, we obtain

$$8f(1) + 6f(1) - 1 = 5$$

$$\Rightarrow \; 14f(1) = 6 \Rightarrow f(1) = \frac{3}{7}$$

Hence $\left.\dfrac{dy}{dx}\right|_{x=1} = 2\times\dfrac{3}{7}+\dfrac{1}{2} = \dfrac{19}{14}$

**12. (c) :** $\dfrac{dy}{dx} = \dfrac{2}{3}x^{-1/3}$. This derivative is never zero, but there is no derivative for $x = 0$. So (0, 0) is a critical point. If $x < 0$ then $\dfrac{dy}{dx} < 0$ and if $x > 0$ then $\dfrac{dy}{dx} >$ 0. Thus (0, 0) is a point of minimum.

**13. (d):** The given function is periodic, with period $2\pi$. So the difference between the greatest and least values of the function is the difference between these values on the interval $[0, 2\pi]$. We have

$$f'(x) = -(\sin x + \sin 2x - \sin 3x)$$

$$= -4\sin x \sin\left(\frac{3x}{2}\right)\sin\left(\frac{x}{2}\right)$$

Hence $x = 0, \dfrac{2\pi}{3}, \pi$ and $2\pi$ are the critical points.

Also, $f(0) = 1 + \dfrac{1}{2}-\dfrac{1}{3} = \dfrac{7}{6}$, $f\left(\dfrac{2\pi}{3}\right) = -\dfrac{13}{12}$,

$$f(\pi) = -\frac{1}{6} \text{ and } f(2\pi) = \frac{7}{6}.$$

Hence the greatest value is $\dfrac{7}{6}$ and the least value is $\left(-\dfrac{13}{12}\right)$.

Hence the difference is $\dfrac{7}{6}-\left(\dfrac{-13}{12}\right) = \dfrac{27}{12} = \dfrac{9}{4}$

**14. (b):** Let $I = \int_0^{\pi/4}\ln(1+\tan^2\theta+2\tan\theta)\,d\theta$

$$= \int_0^{\pi/4}\ln(1+\tan\theta)^2\,d\theta = \int_0^{\pi/4}2\ln(1+\tan\theta)\,d\theta$$

Let $I_1 = \int_0^{\pi/4}\ln(1+\tan\theta)\,d\theta$

$$I_1 = \int_0^{\pi/4}\ln\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)d\theta$$

$$= \int_0^{\pi/4}\ln\left(1+\frac{1-\tan\theta}{1+\tan\theta}\right)d\theta$$

$$= \int_0^{\pi/4}[\ln 2-\ln(1+\tan\theta)]\,d\theta$$

$$\Rightarrow \; 2I_1 = \frac{\pi}{4}\ln 2 = I$$

Hence $\dfrac{\pi \ln a}{b} = \dfrac{\pi}{4}\ln 2$. Hence $a = 2$ and $b = 4$.

So $2a + 3b = 16$

**15. (d):** $\dfrac{dx}{dy} = 2x - y^2$ (This is a linear equation in $x$).

The integrating factor is $e^{-\int 2dy} = e^{-2y}$

So $\dfrac{d}{dy}(xe^{-2y}) = -y^2e^{-2y}$

Integrating, we have

$$xe^{-2y} = \frac{-y^2e^{-2y}}{-2} - \int ye^{-2y}dy + \text{constant}$$

$$= \frac{y^2}{2}e^{-2y} + \frac{ye^{-2y}}{2} - \frac{1}{2}\int e^{-2y}dy + \text{constant}$$

$$= \frac{y^2}{2}e^{-2y} + \frac{y}{2}e^{-2y} + \frac{e^{-2y}}{4} + C$$

$$\therefore \quad x = Ce^{2y} + \frac{y^2}{2} + \frac{y}{2} + \frac{1}{4}$$

**16. (b):** Length of each side of the triangle is 2 so it is an equilateral triangle and hence its incentre is the centroid $\left(-\dfrac{1}{\sqrt{3}}, -1\right)$ of the triangle.

**17. (c) :** Let the equation of the other tangent from the origin be $y = mx$, then length of the perpendiculars from the centre $(2, -1)$ on the two tangents is same.

$$\Rightarrow \left|\frac{2m+1}{\sqrt{1+m^2}}\right| = \left|\frac{6-1}{\sqrt{9+1}}\right| = \frac{5}{\sqrt{10}}$$

$\Rightarrow \; 10(2m + 1)^2 = 25(1 + m^2)$

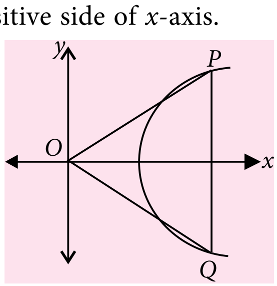
$\Rightarrow \; 3m^2 + 8m - 3 = 0 \Rightarrow (3m - 1)(m + 3) = 0$

$\Rightarrow \; m = -3$ or $1/3$

$m = -3$ represents the given tangent hence the slope of the required tangent is $1/3$ and its equation is $y = (1/3)x$

$\Rightarrow \; x - 3y = 0$

**18. (c) :** As $\angle POQ = 60°$. $OP$ makes an angle of $30°$ with the positive side of $x$-axis.




Equation of $OP$ is $y = \dfrac{1}{\sqrt{3}}x$, which meets the hyperbola at points for which

$$\frac{3y^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow (3b^2 - a^2)y^2 = a^2b^2$$

For real values of $y$, $3b^2 > a^2$

$\Rightarrow \; 3a^2(e^2 - 1) > a^2 \;\Rightarrow\; 3e^2 > 4 \;\Rightarrow\; e > \dfrac{2}{\sqrt{3}}$

**19. (a) :** Equation of the tangent at $(t^2, 2t)$ to the parabola $y^2 = 4x$ is $ty = x + t^2$ ......(1)

And the tangent at $(-16t', -8t'^2)$ to the parabola $x^2 = -32y$ is $t'x = y - 8t'^2$ .......(2)

Since (1) and (2) represent the same line, comparing we get

$$\frac{-1}{-t'} = \frac{t}{1} = \frac{t^2}{8t'^2} \Rightarrow t = 2$$

Hence the required slope of the tangent is $\dfrac{1}{t} = \dfrac{1}{2}$

**20. (c) :** Any point on the given line is $P(2r, 3r + 2, 4r + 3)$ and the given point be $A$

Direction ratios of $AP$ are

$2r - 3, 3r + 2 + 1, 4r + 3 - 11$

or $\; 2r - 3, 3r + 3, 4r - 8$

$AP$ is perpendicular to the given line

if $\; 2(2r - 3) + 3(3r + 3) + 4(4r - 8) = 0$

$\Rightarrow \; r = 1$, so coordinates of point are $(2, 5, 7)$

$\therefore \; AP = \sqrt{(2-3)^2 + (5+1)^2 + (7-11)^2} = \sqrt{53}$ units

**21. (a) :** Since $(2)(1) + (-4)(1) + (1)(2) = 0$ the line is parallel to the plane. It will lie in the plane if the point $(4, 2, k)$ lies on the plane.

$\Rightarrow \; 2(4) - 4(2) + k = 7 \Rightarrow k = 7$

**22. (b):** $N = \sum f_i = k[{}^nC_0 + {}^nC_1 + \ldots\ldots + {}^nC_n]$

$= k(1 + 1)^n = k2^n$

$\sum f_ix_i = k[1.{}^nC_1 + 2\,.\,{}^nC_2 + \ldots\ldots + n\,{}^nC_n]$

$$= k\sum_{r=0}^{n} r.{}^nC_r = k \cdot n \cdot 2^{n-1}$$

Thus $\bar{x} = \dfrac{1}{2^n}(n2^{n-1}) = \dfrac{n}{2}$

**23.** **(b):** The product will be divisible by 5 or 10 if at least one of the number has last digit as 0 or 5.

Thus required probability = 1 – $P$(none of the number has last digit 0 or 5)

$$=1-\left(\frac{8}{10}\right)^4=1-\left(\frac{4}{5}\right)^4=\frac{369}{625}$$

**24.** **(c) :** Let $X$ be the number of heads obtained by $A$ and $Y$ be the number of heads obtained by $B$. Note that both $X$ and $Y$ are binomial variate with parameters $n$ = 3 and $p$ = 1/2.

Probability that both $A$ and $B$ obtain the same number of heads is

$\therefore$ $P(X = 0)\ P(Y = 0) + P(X = 1)\ P(Y = 1) + P(X = 2)\ P(Y = 2) + P(X = 3)\ P(Y = 3))$

$$=\left[{}^3C_0\left(\frac{1}{2}\right)^3\right]^2+\left[{}^3C_1\left(\frac{1}{2}\right)^3\right]^2+\left[{}^3C_2\left(\frac{1}{2}\right)^3\right]^2+\left[{}^3C_3\left(\frac{1}{2}\right)^3\right]^2$$

$$=\left(\frac{1}{2}\right)^6[1+9+9+1]=\frac{20}{64}=\frac{5}{16}$$

**25.** **(c) :** The given expression is equal to

$8(1 + \cos A) - 16(\cos 2A - \cos 3A)$

$= 8(1 + \cos A) - 16[2\cos^2 A - 1 - \cos A(4\cos^2 A - 3)]$

$$=8\left(1+\frac{3}{4}\right)-16\left[2\times\frac{9}{16}-1-\frac{3}{4}\left(4\times\frac{9}{16}-3\right)\right]$$

$= 14 - (18 - 16 - 27 + 36) = 3$

**26.** **(b):** We have $x^2 - 2x + 4 = -3\cos(ax + b)$

$\Rightarrow$ $(x - 1)^2 + 3 = -3\cos(ax + b)$ ...(1)

As $(x - 1)^2 \geq 0$ and $-1 \leq \cos(ax + b) \leq 1$

Equation (1) is possible if $\cos(ax + b) = -1$ and $x - 1 = 0$

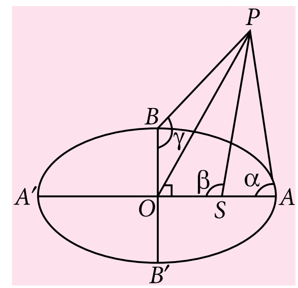
$\Rightarrow$ $a + b = \pi, 3\pi, 5\pi, \ldots\ldots\ldots\ldots$

Since $a + b \leq 6$ and $3\pi > 6$, $a + b = \pi$

Hence the locus of $(a, b)$ is $x + y = \pi$

**27.** **(c) :** Let the equation of the elliptic field be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$

$OP$ be the tower of height $h$ at $O$, the centre of the field, Let $A$ and $B$ be extremities of major and minor axis respectively and $S$ be a focus, then




$OA = h\cot\alpha = a$

$OS = h\cot\beta = ae$

$OB = h\cot\gamma = b$

Since $b^2 = a^2(1 - e^2)$

$\Rightarrow$ $\cot^2\gamma = \cot^2\alpha - \cot^2\beta$

**28.** **(c) :** Table for basic logical connectives :

| $p$ | $q$ | $\sim q$ | $p \leftrightarrow \sim q$ | $\sim(p \leftrightarrow \sim q)$ | $(p \leftrightarrow q)$ |
|---|---|---|---|---|---|
| T | T | F | F | T | T |
| T | F | T | T | F | F |
| F | T | F | T | F | F |
| F | F | T | F | T | T |

Note that $\sim(p \leftrightarrow \sim q)$ is not a tautology.

$\therefore$ Statement – II is false.

From table $\sim(p \leftrightarrow q)$ is equivalent to $(p \leftrightarrow q)$

Thus, statement-I is true.

**29.** **(b):** Mid point of the segment $AB$ is

$\left(\frac{1+1}{2},\frac{0+6}{2}+\frac{7+3}{2}\right)$ = (1, 3, 5) which lies on the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ So, Statement II is true.

Direction ratios of $AB$ are 0, 6, – 4 and the given line are 1, 2, 3.

Since $0 \times 1 + 6 \times 2 + (-4) \times 3 = 0$, the line $AB$ is perpendicular to the given line so, statement I is also true, but statement II is not a correct explanation for Statement I.

**30.** **(a):** Statement II is true.

We have,

$\bar{z}=a+\overline{(b+ic)}^{2017}+\overline{(b-ic)}^{2017}$

$= a + (b - ic)^{2017} + (b + ic)^{2017}$

$= z$

$\Rightarrow$ $z$ is real

$\therefore$ Statement I is true and Statement II is correct explanation for it.

# Math Archives 10 Best Problems

Math Archives, as the title itself suggests, is a collection of various challenging problems related to the topics of JEE (Main & Advanced) Syllabus. This section is basically aimed at providing an extra insight and knowledge to the candidates preparing for JEE (Main & Advanced). In every issue of MT, challenging problems are offered with detailed solution. The readers' comments and suggestions regarding the problems and solutions offered are always welcome.

**1.** The equation of the tangent line of slope 1 to the curve $f(x)=\int_{-1}^{x}|t|\,dt$ is

(a) $y = x + 2$ (b) $y = x - 2$

(c) $y = x - 1$ (d) none of these

**2.** If $I_1=\int_0^1\frac{1+x^8}{1+x^4}dx$ and $I_2=\int_0^1\frac{1+x^9}{1+x^3}dx$, then

(a) $I_1 > 1, I_2 < 1$ (b) $I_1 < 1, I_2 > 1$

(c) $1 < I_1 < I_2$ (d) $I_2 < I_1 < 1$

**3.** Which of the following is not correct ?

(a) $\int_{-a}^{a} x\cdot(f(\cos x))^2\cdot dx=0$

(b) $\int_0^{n\pi} f(\cos^2 x)\,dx=n\int_0^{\pi} f(\cos^2 x)\,dx, n\in N$

(c) $\int_0^{b-c} f(x+c)\,dx=\int_b^c f(x)\,dx$

(d) $\int_a^{\pi-a} x.f(\sin x)\,dx=\frac{\pi}{2}\int_a^{\pi-a} f(\sin x)\,dx$

**4.** $\int_{-1}^{1}\frac{d}{dx}\left(\frac{1}{1+e^{1/x}}\right)dx$ is equal to

(a) $\frac{e}{e+1}$ (b) $\frac{1}{e+1}$

(c) $\frac{2}{1+e}$ (d) none of these

**5.** The function $L(x)=\int_1^x\frac{dt}{t}$ satisfies the equation

(a) $L(x + y) = L(x) + L(y)$

(b) $L(x/y) = L(x) + L(y)$

(c) $L(xy) = L(x) + L(y)$

(d) None of these

**6.** The value of $\int_0^1 e^{x^2}dx$ is

(a) less than $e - 1$ (b) greater than $e$

(c) less than $e - e^2$ (d) $1 - e$

**7.** A differential equation is called linear, if its

(a) degree is 1 (b) order is 1

(c) degree and order both are 1

(d) none of these

**8.** The orthogonal trajectories of a family of parallel lines is a family of

(a) parallel lines (b) concurrent lines

(c) concentric circles (d) concentric ellipses

**9.** Let $f$ be a twice differentiable function such that $f''(x) = -f(x)$ and $f'(x) = g(x)$ if $h(x) = (f(x))^2 + (g(x))^2$ and $h(5) = 2$, then $h(10) =$

(a) 0 (b) 1

(c) 2 (d) none of these

**10.** If the tangent to the curve $xy + ax + by = 0$ at (1, 1) makes an angle $\tan^{-1}2$ with $x$-axis, then $\frac{a+b}{ab}=$

(a) 0 (b) 1/2

(c) −1/2 (d) None of these

## SOLUTIONS

**1.** **(d):** $f'(x) = |x| = 1 \Rightarrow x = -1, 1$

$f(-1) = 0,\ f(1)=\int_{-1}^{1}|t|\,dt=1.$

Thus required tangents have points of contact as (−1, 0), (1, 1).

Tangents are $y = x + 1$, $y - 1 = x - 1 \Rightarrow y = x + 1$ and $y = x$.

**2.** **(d):** Obviously $I_1, I_2 < 1$, as $\frac{1+x^8}{1+x^4}<1, \forall x\in(0,1)$

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and $\dfrac{1+x^9}{1+x^3} < 1,\ \forall x \in (0, 1)$.

Now $1 + x^8 > 1 + x^9$ and $1 + x^4 < 1 + x^3$, $\forall\ x \in\ (0, 1)$

Thus $\dfrac{1+x^8}{1+x^4} > \dfrac{1+x^9}{1+x^3},\ \forall\ x \in (0, 1) \Rightarrow I_1 > I_2$.

**3. (c) :** Note that $\int\limits_0^{b-c} f(x+c)\,dx = -\int\limits_b^c f(x)\,dx$

**4. (d):** The given integral $= \left[\dfrac{1}{1+e^{1/x}}\right]_{-1}^{1}$

$= \dfrac{1}{1+e} - \dfrac{e}{1+e} = \dfrac{1-e}{1+e}$

**5. (c) :** $L(x) = \ln|x|$

$\Rightarrow\ L(xy) = \ln|xy| = \ln|x|\,|y| = \ln|x| + \ln|y|$

$= L(x) + L(y)$

**6. (a) :** $e^{x^2} < e^x \Rightarrow \int\limits_0^1 e^{x^2}dx < \int\limits_0^1 e^x dx = e - 1$

**7. (d):** A differential equation is called linear if its degree is 1 and $y, \dfrac{dy}{dx}, \dfrac{d^2y}{dx^2}, .....$ are not multiplied among themselves.

**8. (a) :** If $y = mx + c$ is a family of parallel lines ($m$ being fixed) then orthogonal trajectory is $y = -\dfrac{1}{m}x + c_1$, which is again a family of parallel lines.

**9. (c) :** $g(x) = f'(x)$

$\therefore\ g'(x) = f''(x) = -f(x)$

$h'(x) = 2f(x)\,f'(x) + 2g(x)\,g'(x)$

$= 2f(x)\,f'(x) + 2f'(x)\cdot\{-f(x)\} = 0$

$\therefore\ h(x) = \text{constant} = 2.$

**10. (b):** Given curve is $xy + ax + by = 0$ ...(1)

$\therefore\ y + x\dfrac{dy}{dx} + a + b\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx} = \dfrac{-a+y}{b+x}$

$\dfrac{dy}{dx} = -\dfrac{a+1}{b+1} = 2$ at $(1, 1)$ $\therefore\ a + 2b = -3$

Also $(1, 1)$ lies on (1)

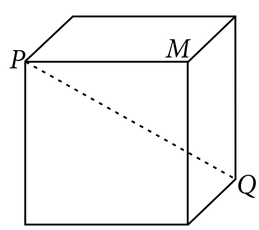
$\therefore\ a + b = -1$

$b = -2,\ a = 1$

$\Rightarrow\ \dfrac{a+b}{ab} = \dfrac{-1}{-2} = \dfrac{1}{2}$

# OLYMPIAD CORNER

1. If $m$ and $n$ are positive integers such that $\dfrac{m+n}{m^2+mn+n^2}=\dfrac{4}{49}$ then $m+n$ must be
2. An equilateral triangle of side length 2 units is inscribed in a circle. The length of a chord of this circle which passes through the midpoints of two sides of this triangle is
3. How many ways are there of walking up a flight of 10 stairs if you take either one or three stairs with each step?
4. A straight line joins two opposite vertices $P$ and $Q$ of a cube of side length one metre and $M$ is any other vertex. What is the distance, in metres, from $M$ to the closest point on the line $PQ$?




5. In a soccer tournament eight teams play each other once, with two points awarded for a win, one point for a draw and zero for a loss. How many points must a team score to ensure that it is in the top four (*i.e.* has more points than at least four other teams)?

## SOLUTIONS

**1.** Assume that $m+n=4k$, $m^2+mn+n^2=49k$. Then

$m^2+2mn+n^2=(m+n)^2=(4k)^2=16k^2$,

hence $mn = 16k^2 - 49k$. Since $mn > 0$, from $16k^2 - 49k > 0$ we find that $k > 3$. Since we also have the identity

$$mn=\left(\frac{m+n}{2}\right)^2-\left(\frac{m-n}{2}\right)^2$$

$$\left(\frac{m-n}{2}\right)^2\geq 0,$$

we also find that

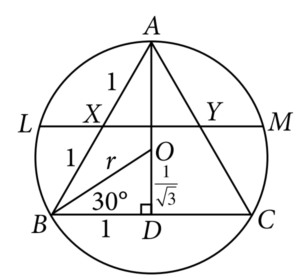
$$0\leq\left(\frac{m+n}{2}\right)^2-mn=\left(\frac{4k}{2}\right)^2-\left(16k^2-49k\right)$$

$=49k-12k^2$, from which $k \leq 4$ follows.

Hence $k = 4$ (since it must be an integer), and from $m+n=16$ and $mn=6$, $n=10$, and $m^2+mn+n^2=196$ follow. Indeed,

$$\frac{m+n}{m^2+mn+n^2}=\frac{16}{196}=\frac{4}{49}.$$

**2.**




Let the circumcircle of the equilateral $\Delta ABC$ have centre $O$, (0, 0) and radius $r$, Join $A$ to $D$, the mid-point of $BC$, then $AD$ passes through $O$ and is perpendicular to $BC$. Draw $OB$. Let the chord $LM$ cut the sides $AB$ and $AC$ of $\Delta ABC$ at $X$ and $Y$. Then $LXYM$ is parallel to $BC$.

$OD = \tan 30° = \frac{1}{\sqrt{3}}$

Then $D = \left(0, -\frac{1}{\sqrt{3}}\right)$,

$A = \left(0, \frac{2}{\sqrt{3}}\right)$ and

$B = \left(-1, -\frac{1}{\sqrt{3}}\right)$ and the equation of the circle is $x^2 + y^2 = \frac{4}{3}$.

Now $X$ is the mid-point of $AB$ so

$$X = \left(-\frac{1}{2}, \frac{1}{2}\left(\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right)\right) = \left(-\frac{1}{2}, -\frac{1}{2\sqrt{3}}\right)$$

To find the $x$ – coordinates of $L$ and $M$, substitute the $y$ - coordinate of $X$ in the equation $x^2 + y^2 = \frac{4}{3}$, *i.e.*

$$x^2 + \left(\frac{1}{2\sqrt{3}}\right)^2 = \frac{4}{3}$$

$$\Rightarrow x^2 = \frac{4}{3} - \frac{1}{12} = \frac{5}{4}$$

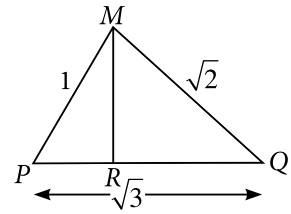
$$\Rightarrow x = \pm\frac{\sqrt{5}}{2}$$

Thus the $x$-coordinate of $L$ is $-\frac{\sqrt{5}}{2}$ and the $x$-coordinate of $M$ is $\frac{\sqrt{5}}{2}$, so the length of $LM$ is $\frac{\sqrt{5}}{2} + \frac{\sqrt{5}}{2} = \sqrt{5}$.

**3.** The problem is equivalent to that of writing 10 as a sum of a 1s and 3s, for example, $10 = 1 + 3 + 1 + 3 + 1 + 1$.

| No. of 3s | No. of 1s | Pattern | Number of Expressions |
|---|---|---|---|
| 0 | 10 | 1111111111; | 1 |
| 1 | 7 | 31111111, 13111111, ..., 11111113; | 8 |
| 2 | 4 | 331111, 133111, ..., 111133; | 5 |
| | | 313111, 131311, 113131, 111313; | 4 |
| | | 311311, 131131, 113113; | 3 |
| | | 311131, 131113; | 2 |
| | | 311113; | 1 |
| 3 | 1 | 1333, 3133, 3313, 3331; | 4 |
| | | Total | 28 |

**4.** If we draw the triangle $MPQ$ we see that its sides are $1, \sqrt{2}$ and $\sqrt{3}$, and is right angled at $M$. We need to know the length of the altitude $MR$. Since triangles $MPQ$ and $RPM$ are similar, it can be seen that $\frac{MR}{\sqrt{2}} = \frac{1}{\sqrt{3}}$, so $MR = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$.




**5.** Since there are 8 teams, there are 7 rounds of four matches and thus a total of $7 \times 8 = 56$ points available.

Consider a team with 10 points. It is possible to have 5 teams on 10 points and 3 teams on 2 points when each of the top 5 draws with each other, each of the bottom 3 draws with each other and each of the top 5 wins against each of the bottom 3. So 10 points does not guarantee a place in the top 4.

Consider a team with 11 points. If this team was fifth then the number of points gained by the top 5 teams is $\geq 55$. This is impossible as the number of points shared by the bottom 3 teams is then 1, as these 3 teams must have at least $3 \times 2 = 6$ points between them for the games played between themselves. Hence 11 points is sufficient to ensure a place in the top 4. Thus 11 points are required.


**Form IV**

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Mahabir Singh
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# PRACTICE PAPER 2016
# JEE MAIN

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1. Let $A=\begin{bmatrix}-4 & 3\\ -7 & 5\end{bmatrix}$ then $A^{482}$, $A^{700}$, $A^{345}$ are respectively
(a) $A - I, A, -A + I$ (b) $A, -A, I$
(c) $A - I, -A, -I$ (d) $A - I, -A + I, -I$

2. If $S_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$, $t_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$, then $\frac{t_n}{S_n} =$
(a) $n/2$ (b) $n$ (c) $2n$ (d) $n/3$

3. If $C_0$, $C_1$, ..... are the binomial coefficients in the expansion of $(1 + x)^n$, and $\sum_{r=0}^{n} (-1)^r \frac{C_r}{(r+1)^2} = \sum_{r=0}^{n} \frac{1}{r+1}$, then $k$ is equal to
(a) $\frac{1}{n}$ (b) $\frac{1}{n+1}$
(c) $\frac{n}{n+1}$ (d) none of these

4. Assume that $f$ is continuous on $[a, b]$, $a > 0$ and differentiable on an open interval $(a, b)$. If $\frac{f(a)}{a} = \frac{f(b)}{b}$, then there exist $x_0 \in (a, b)$ such that
(a) $x_0 f'(x_0) = f(x_0)$ (b) $f'(x_0) + x_0 f(x_0) = 0$
(c) $x_0 f'(x_0) + f(x_0) = 0$
(d) $f'(x_0) = x_0^2 f(x_0)$

5. Number of ordered pairs of real numbers such that $(a + ib)^{2008} = (a - ib)$ holds good, is
(a) 2007 (b) 2008 (c) 2009 (d) 2010

6. For a curve $y = f(x)$, $f''(x) = 4x$ at each point $(x, y)$ on it and it crosses the $x$-axis at $(-2, 0)$ an angle of 45° with positive direction of $x$-axis. If $f(x) = ax^3 + bx^2 + cx + d$ then the value of $3(a + b + c + d)$ equals
(a) 45 (b) $-15$ (c) $-45$ (d) none of these

7. $\sum_{K=1}^{10} \frac{(-1)^{K-1}}{K} . ({}^{10}C_K) =$
(a) $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+....+\frac{1}{11}$
(b) $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+....+\frac{1}{10}$
(c) $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+.....+\frac{1}{9}$
(d) $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+.....+\frac{1}{12}$

8. The greatest and the least value of $|z_1 + z_2|$ if $z_1 = 24 + 7i$ and $|z_2| = 6$ are respectively
(a) 31, 19 (b) 25, 19
(c) 31, 25 (d) none of these

9. If the straight lines $\frac{x-1}{K} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x-2}{3} = \frac{y-3}{K} = \frac{z-1}{2}$ intersect at a point then the integer $K=$
(a) $-2$ (b) $-5$ (c) 5 (d) 2

10. Let $f(x) = \min\{1, \cos x, 1 - \sin x\}$ $-\pi \le x \le \pi$ then
(a) $f(x)$ is derivable at $x = 0$
(b) $f(x)$ has local maximum at $x = 0$
(c) $f(x)$ is derivable at $x = \pi/2$
(d) none of these

11. The remainder when $(2222)^{5555}$ is divided by 7 is
(a) 2 (b) 3 (c) 4 (d) 5

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12. It is given that complex numbers $z_1$ and $z_2$ satisfy $|z_1| = 2$ and $|z_2| = 3$. If the included angle of their corresponding vectors is 60° then $\left|\dfrac{z_1+z_2}{z_1-z_2}\right|$ can be expressed as $\dfrac{\sqrt{x}}{7}$ where '$x$' is a natural number then '$x$' equals to
(a) 19 (b) 119 (c) 126 (d) 133

13. Let $p$ and $q$ are positive integers. $f$ is a function defined for positive numbers and attains only positive values such that $f(xf(y)) = x^p y^q$ then
(a) $q = p^2$ (b) $p = q^2$
(c) $p = q$ (d) $p = 2q$

14. In a $\Delta ABC$, $AB = AC$, $P$ and $Q$ are points on $AC$ and $AB$ respectively such that $CB = BP = PQ = QA$. If $\angle AQP = \theta$, then $\tan^2\theta$ is a root of the equation
(a) $y^3 + 21y^2 - 35y - 12 = 0$
(b) $y^3 - 21y^2 + 35y - 12 = 0$
(c) $y^3 - 21y^2 + 35y - 7 = 0$
(d) $12y^3 - 35y^2 + 35y - 12 = 0$

15. Consider a family of circles which are passing through the point (–1, 1) and are tangent to $x$-axis. If $(h, k)$ is the centre of the circles, then
(a) $-\frac{1}{2} \le k \le \frac{1}{2}$ (b) $k \le \frac{1}{2}$
(c) $0 \le k \le \frac{1}{2}$ (d) $k \ge \frac{1}{2}$

16. Direction cosine of normal to the plane containing lines $x = y = z$ and $x-1 = y-1 = \dfrac{z-1}{d}$ (where $d \in R - \{1\}$), are
(a) $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$ (b) $\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$
(c) $\left(0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (d) none of these

17. If $f(x)$ satisfies the relation
$$\int_{-2}^{x} f(t)dt + x f'''(3) = \int_{1}^{x} x^3 dx + f'(1)\int_{0}^{x} x^2 dx + f''(2)\int_{3}^{x} x\, dx$$
then $f(x)$ =
(a) $x^3 + 5x^2 + 3x + 6$ (b) $x^3 + 5x^2 - 2x - 6$
(c) $x^3 - 5x^2 + 2x - 6$ (d) $2x^3 - 5x^2 - 2x + 6$

18. If the lengths of medians of a triangle are 2 units, 3 units and 4 units, then the area of the triangle is
(a) greater than $4\sqrt{3}$ (b) less than $4\sqrt{3}$
(c) less than or equal to $4\sqrt{3}$
(d) greater than or equal to $4\sqrt{3}$

19. Consider a curve $ax^2 + 2hxy + by^2 = 1$ and a point $P$ not on the curve. A line drawn from $P$ intersects the curve at points $Q$ and $R$. If the product $PQ \cdot PR$ is independent of the slope of the line, then the curve is
(a) parabola (b) circle
(c) ellipse (d) hyperbola

20. Let $A = \begin{bmatrix} 1 & -2 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$ by observing orthogonally among the column vectors of $A$ one may obtain the inverse of $A$ as
(a) $A = \begin{bmatrix} 1 & 1 & 1 \\ -2 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{2}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$
(c) $\begin{bmatrix} 2 & 2 & 2 \\ -2 & 1 & 1 \\ 0 & -3 & 3 \end{bmatrix}$ (d) $\frac{1}{6}\begin{bmatrix} 2 & -2 & 0 \\ 2 & 1 & 3 \\ 2 & 1 & -3 \end{bmatrix}$

21. The number of rational terms in the expansion of $\left(\sqrt{2} + \sqrt[3]{3} + \sqrt[6]{5}\right)^{10}$ is
(a) 2 (b) 3 (c) 4 (d) 0

22. Number of ways of giving away 10 different gifts to 5 students so that each get at least one and a particular student gets at least 5 gifts is
(a) 5040 (b) 60480
(c) 65520 (d) 10080

23. The shortest distance between the lines
$\vec{r} = 3\hat{i} - 15\hat{j} + 9\hat{k} + \lambda(2\hat{i} - 7\hat{j} + 5\hat{k})$ and
$\vec{r} = (-\hat{i} + \hat{j} + 9\hat{k}) + \mu(2\hat{i} + \hat{j} - 3\hat{k})$ is
(a) $\sqrt{34}$ (b) $\sqrt{3}$ (c) $3\sqrt{3}$ (d) $4\sqrt{3}$

24. How many real solutions does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have ?
(a) 5 (b) 7 (c) 1 (d) 3

25. Suppose $S=\frac{1}{n^4}\prod_{r=1}^{2n}(n^2+r^2)^{1/n}$, then $\lim_{n\to\infty}\log_e S$ is equal to

(a) $\int_0^2 \log_e(2+x^2)dx$ (b) $\int_0^1 \log_e(1+x^2)dx$

(c) $2\int_0^1 \log_e(1+x^2)dx$ (d) $\int_0^2 \log_e(x^2-4x+5)dx$

26. The number of ordered pairs of positive integers $(a, b)$ such that LCM of $a$ and $b$ is $2^3 5^7 11^{13}$ is

(a) 2385 (b) 2835

(c) 3825 (d) 8325

27. If $a^2x^4 + b^2y^4 = c^6$, then the greatest value of $xy$ is

(a) $\frac{c^2}{\sqrt{2ab}}$ (b) $\frac{c^3}{\sqrt{2ab}}$

(c) $\frac{c^4}{\sqrt{2ab}}$ (d) $\frac{c^5}{\sqrt{2ab}}$

28. The equation $\sin x + x\cos x = 0$ has at least one root in

(a) $\left(-\frac{\pi}{2}, 0\right)$ (b) $(0, \pi)$

(c) $\left(\pi, \frac{3\pi}{2}\right)$ (d) $\left(0, \frac{\pi}{2}\right)$

29. From a point $(h, k)$ three normals are drawn to the parabola $y^2 = 4ax$. Tangents are drawn to the parabola at the feet of the normals to form a triangle. Then the centroid of the triangle is

(a) $\left(\frac{2a-h}{3}, -\frac{k}{2}\right)$ (b) $(a, -k)$

(c) $\left(\frac{3a-h}{2}, 0\right)$ (d) $\left(\frac{2a-h}{3}, 0\right)$

30. If $z+\frac{1}{z}=\sqrt{3}$, then $\sum_{r=1}^{5}\left(z^r+\frac{1}{z^r}\right)^2 =$

(a) 8 (b) 10 (c) 12 (d) 15

31. For a twice differentiable function $f(x)$, $g(x)$ is defined as $g(x) = (f'(x))^2 + f(x)f''(x)$ on $(a, e)$. If $a < b < c < d < e, f(a) = 0, f(b) = 2, f(c) = -1, f(d) = 2,$ $f(e) = 0$, then the minimum number of roots of the equation $g(x) = 0$ is

(a) 4 (b) 5 (c) 6 (d) 7

32. If $A$, $B$, $C$ are angles in a $\Delta ABC$ and $\sin\left(A-\frac{\pi}{4}\right)\sin\left(B-\frac{\pi}{4}\right)\sin\left(C-\frac{\pi}{4}\right)=\frac{1}{2\sqrt{2}}$, then $\Sigma \tan A \tan B =$

(a) $\Sigma \tan A$ (b) $\Sigma \cot A$

(c) $(\Sigma \tan A)^{-1}$ (d) $(\Sigma \cot A)^{-1}$

33. If $\int_0^{\pi/2}\frac{dx}{1+\sin x+\cos x}=\log 2$, then the value of $\int_0^{\pi/2}\frac{\sin x}{1+\sin x+\cos x}dx$ is equal to

(a) $\frac{1}{2}\log 2$ (b) $\frac{\pi}{2}-\log 2$

(c) $\frac{\pi}{4}-\frac{1}{2}\log 2$ (d) $\frac{\pi}{2}+\log 2$

34. If $\lim_{x\to\infty}\frac{4x^2+3x+5}{x+1+x^{k-1}}$ exists then

(a) $k = 2$ (b) $k < 2$ (c) $k \geq 3$ (d) $k < -3$

35. $OA$ is the perpendicular drawn from the centre '$O$' of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, to the tangent at any point $P$ on the ellipse. If the normal to the ellipse at the point $P$ meets the $X$-axis at $B$, then $(OA)\cdot(PB)$ is

(a) $a^2$ (b) $a\sqrt{a^2+b^2}$

(c) $b^2$ (d) $b\sqrt{a^2+b^2}$

36. If three distinct normals can be drawn to the parabola $y^2 - 2y = 4x - 9$ from the point $(2a, 0)$ then range of values of $a$ is

(a) no real values possible

(b) $(2, \infty)$ (c) $(-\infty, 2)$

(d) none of these

37. The determinant $\begin{vmatrix} xp+y & x & y \\ yp+z & y & z \\ 0 & xp+y & yp+z \end{vmatrix}=0$, if

(a) $x, y, z$ are in A.P. (b) $x, y, z$ are in G.P

(c) $x, y, z$ are in H.P. (d) $xy, yz, zx$ are in A.P.

38. A function $g(x)$ is continuous in $[0, \infty)$ satisfying $g(1) = 1$ and if $\int_0^x 2xg^2(t)dt=\left(\int_0^x 2g(x-t)dt\right)^2$, then $g(x)$ is

(a) $\sqrt{x}$ (b) $x^{1/\sqrt{2}}$

(c) $x$ (d) $x^{1+\sqrt{2}}$

39. Let $f(x) = \cos^2 x + \cos^2 2x + \cos^2 3x$. Number of values of $x \in [0, 2\pi]$ for which $f(x)$ equals the smallest positive integer is
(a) 3 (b) 4 (c) 5 (d) none of these

40. The radius of the largest circle with centre at $(a, 0)$, $(a > 0)$ that can be inscribed in the ellipse $x^2 + 4y^2 = 16a^2$ is
(a) $\frac{11a}{\sqrt{3}}$ (b) $\sqrt{\frac{11}{3}}a$
(c) $\frac{\sqrt{11}}{3}a$ (d) $\frac{11}{3}a$

41. If $\int (2-3\sin^2 x)\sqrt{\sec x}\,dx = 2f(x)\sqrt{g(x)} + c$ and $f(x)$ is non constant function, then
(a) $f^2(x) + g^2(x) = 1$ (b) $f^2(x) - g^2(x) = 1$
(c) $f(x)\,g(x) = 1$ (d) $f(x) = g(x)$

42. Three distinct vertices are randomly chosen among the vertices of a cube. The probability that they are vertices of an equilateral triangle is
(a) 3/7 (b) 4/7 (c) 1/7 (d) 6/7

43. The probability that a randomly chosen positive divisor of $10^{99}$ is an integer multiple of $10^{88}$ is
(a) $\left(\frac{3}{25}\right)^2$ (b) $\left(\frac{3}{5}\right)^2$
(c) $\left(\frac{8}{9}\right)^2$ (d) $\left(\frac{8}{9}\right)^4$

44. A license plate consists of 8 digits out of 10 digits 0, 1, 2, 3, ...., 9. It is called even if it contain an even number of 0's.The number of even license plates is
(a) $\frac{10^8 - 8^8}{2}$ (b) $10^8 - 8^{10}$
(c) $\frac{10^8 + 8^8}{2}$ (d) $\frac{10^8 + 8^{10}}{2}$

45. The shape of surface of a curved mirror such that light from a source at origin will be reflected in a beam of rays parallel to $x$-axis is
(a) circle (b) parabola
(c) ellipse (d) hyperbola

46. The area of the loop of the curve $y^2 = x^4(x + 2)$ is [in square units ]
(a) $\frac{32\sqrt{2}}{105}$ (b) $\frac{64\sqrt{2}}{105}$
(c) $\frac{128\sqrt{2}}{105}$ (d) $\frac{256\sqrt{2}}{105}$

47. If $n \in N$, then the remainder when $37^{n+2} + 16^{n+1} + 30^n$ is divided by 7 is
(a) 0 (b) 1 (c) 2 (d) 5

48. Let $P$ and $Q$ be the respective intersections of the internal and external angle bisectors of the triangle $ABC$ at $C$ and the side $AB$ produced. If $CP = CQ$, then the value of $(a^2 + b^2)$ is (where $a$ and $b$ and $R$ have their usual meanings for $\Delta ABC$)
(a) $2R^2$ (b) $2\sqrt{2}R^2$
(c) $4R^2$ (d) $4\sqrt{2}R^2$

49. The point of intersection of common tangents of $y^2 = 4ax$ $(a > 0)$ and $x^2 + y^2 - 6ax + a^2 = 0$ is
(a) $(a, 0)$ (b) $(-a, 0)$
(c) $(-a, -a)$ (d) $(-2a, 0)$

50. Let $f(x) \geq 0\ \forall\ x \geq 0$ be a non-negative continuous function. If $f'(x)\cos x \leq f(x)\sin x\ \forall\ x \geq 0$, then the value of $f\left(\frac{5\pi}{3}\right)$ is
(a) $e^{\frac{-1}{\sqrt{3}}}$ (b) $\frac{\sqrt{3}-1}{2}$
(c) $\frac{\sqrt{3}+1}{2}$ (d) 0

## SOLUTIONS

1. (c): Using characteristic equation

$$\begin{vmatrix} -4-\lambda & 3 \\ -7 & 5-\lambda \end{vmatrix} = 0 \Rightarrow -20 - \lambda + \lambda^2 + 21 = 0$$

$\Rightarrow \lambda^2 - \lambda + I = 0$
$\Rightarrow A^2 - A + I = 0$
$\Rightarrow A^3 = -I$
So, $A^{482} = (A^3)^{160} A^2 = (-I)^{160} (A - I) = A - I$
$A^{700} = (A^3)^{233} \cdot A = -A$
$A^{345} = (A^3)^{115} = -I$

2. (a) : $S_n = \sum_{r=0}^{n} \frac{1}{{}^nC_{n-r}}$

$$\Rightarrow nS_n = \sum_{r=0}^{n} \frac{n}{{}^nC_{n-r}} = \sum_{r=0}^{n} \left[\frac{n-r}{{}^nC_{n-r}} + \frac{r}{{}^nC_{n-r}}\right]$$

$$\Rightarrow \frac{t_n}{S_n} = \frac{n}{2}$$

3. (b) : We have $(1+x)^n = \sum_{r=0}^{n} C_r x^r$

$$\Rightarrow \sum_{r=0}^{n} C_r \frac{x^{r+1}}{r+1} = \frac{(1+x)^{n+1} - 1}{(n+1)}$$

$$\Rightarrow \sum_{r=0}^{n} C_r \frac{x^r}{r+1} = \frac{1}{n+1}\left(\frac{(1+x)^{n+1} - 1}{(1+x) - 1}\right)$$

$$\Rightarrow \sum_{r=0}^{n} \frac{C_r}{(r+1)} \int_{-1}^{0} x^r dx$$

$$= \frac{1}{n+1}\left[\int_{-1}^{0} \left((1+x)+(1+x)^2+\ldots+(1+x)^{n-1}\right) dx\right]$$

$$= \frac{1}{n+1} \sum_{r=0}^{n} \int_{-1}^{0} (1+x)^r dx$$

$$\Rightarrow \sum_{r=0}^{n} \frac{(-1)^r C_r}{(r+1)^2} = \frac{1}{n+1} \sum_{r=0}^{n} \frac{1}{r+1}$$

**4.** **(a) :** Consider a function $g(x) = \dfrac{f(x)}{x}$

as $f(x)$ and $x$ are differentiable hence $g(x)$ is also differentiable.

Now, $g(a) = \dfrac{f(a)}{a}$ and $g(b) = \dfrac{f(b)}{b}$

Since, $\dfrac{f(a)}{a} = \dfrac{f(b)}{b}$

$\therefore$ $g(a) = g(b)$

Hence Rolle's theorem is applicable for $g(x)$

$\therefore$ $\exists$ some $x_0 \in (a, b)$

Where $g'(x) = 0$

But $g'(x) = \dfrac{xf'(x) - f(x)}{x^2}$; $g'(x_0) = \dfrac{x_0 f'(x_0) - f(x_0)}{x_0^2} = 0$

$x_0 f'(x_0) = f(x_0)$

**5.** **(d) :** Let $z = a + ib \Rightarrow \bar{z} = a - ib$, hence $z^{2008} = \bar{z}$

$|z|^{2008} = |\bar{z}| = |z| \Rightarrow |z|(|z|^{2007} - 1) = 0$

$\Rightarrow$ $|z| = 0$ or $|z| = 1$

If $|z| = 0 \Rightarrow z = 0 \Rightarrow (0, 0)$

If $|z| = 1$, $z^{2009} = z\bar{z} = |z|^2 = 1 \Rightarrow$ 2009 values of '$z$'

$\therefore$ Total values of '$z$' are 2010

**6.** **(c):** $f''(x) = +4x$

$f'(x) = +2x^2 + c$

$1 = +8 + c_1$ (slope is 45°)

$\Rightarrow c_1 = -7$

$\therefore$ $f'(x) = +2x^2 - 7$

$f(x) = +\dfrac{2x^3}{3} - 7x + c_2$ (passes through (–2, 0))

$\Rightarrow c_2 = -\dfrac{26}{3}$

$f(x) = +\dfrac{2x^3}{3} - 7x - \dfrac{26}{3}$

and given $f(x) = ax^3 + bx^2 + cx + d$

$\Rightarrow a = +\dfrac{2}{3}, b = 0, c = -7, d = -\dfrac{26}{3}$

$\therefore$ $3(a+b+c+d) = 3\left(\dfrac{2}{3} + 0 - 7 - \dfrac{26}{3}\right) = -45$

**7.** **(b) :** Required value is

$$\frac{{}^{10}C_1}{1} - \frac{{}^{10}C_2}{2} + \frac{{}^{10}C_3}{3} \ldots\ldots \frac{{}^{10}C_{10}}{10}$$

to find which, consider

$(1 - x)^{10} = {}^{10}C_0 - {}^{10}C_1 x + {}^{10}C_2 x^2 \ldots. + {}^{10}C_{10} x^{10}$

$$\Rightarrow \frac{(1-x)^{10} - 1}{x} = -[{}^{10}C_1 - {}^{10}C_2 x + \ldots\ldots - {}^{10}C_{10} x^9]$$

$$\Rightarrow \int_0^1 \frac{1-(1-x)^{10}}{x} dx = \int_0^1 [{}^{10}C_1 - {}^{10}C_2 x + \ldots. - {}^{10}C_{10} x^9] dx$$

$$= {}^{10}C_1 - \frac{{}^{10}C_2}{2} + \frac{{}^{10}C_3}{3} - \ldots. - \frac{{}^{10}C_{10}}{10}$$

To find L.H.S. consider

$$I_n = \int_0^1 \frac{1-(1-x)^n}{x} dx \Rightarrow I_{n+1} - I_n = \int_0^1 (1-x)^n dx = \frac{1}{n+1}$$

$$\therefore I_{n+1} = \frac{1}{n+1} + I_n$$

$$\therefore I_{10} = \int_0^1 \frac{1-(1-x)^{10}}{x} dx = \frac{1}{10} + I_9 = \frac{1}{10} + \frac{1}{9} + I_8 \approx \ldots..$$

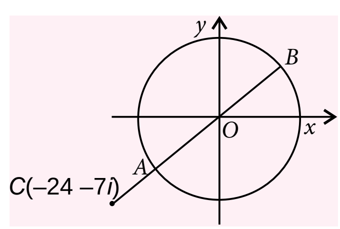
$$= \frac{1}{10} + \frac{1}{9} + \frac{1}{8} + \ldots\ldots + 1$$

**8.** **(a) :** $|z_1 + z_2| = |z_2 - (-24 - 7i)|$

$|z| = 6$, $C(-24 - 7i)$

$OC = 25$, $CA = OC - OA = 25 - 6 = 19$

and $CB = OC + OB = 25 + 6 = 31$




**9.** **(b) :** The lines intersect $\Rightarrow$ they are coplanar

$$\Rightarrow \begin{vmatrix} 1 & 1 & -2 \\ k & 2 & 3 \\ 3 & k & 2 \end{vmatrix} = 0$$

$\Rightarrow$ $1(4 - 3k) - 1(2k - 9) - 2(k^2 - 6) = 0$

$\Rightarrow$ $2k^2 + 5k - 25 = 0 \Rightarrow 2k^2 + 10k - 5k - 25 = 0$

$\Rightarrow$ $2k(k + 5) - 5(k + 5) = 0$

$k = -5, \dfrac{5}{2}$

**10. (b) :** $f(x)=\begin{cases}\cos x & -\pi\le x\le 0\\ 1-\sin x & 0<x<\pi/2\\ \cos x & \pi/2\le x\le\pi\end{cases}$

$$f'(x)=\begin{cases}-\sin x & -\pi\le x\le 0\\ -\cos x & 0<x<\pi/2\\ -\sin x & \pi/2\le x\le\pi\end{cases}$$

$Lf'(0) = 0$, $Rf'(0) = -1 \Rightarrow f'(0)$ does not exist
$Lf'(\pi/2) = 0$, $Rf'(\pi/2) = -1 \Rightarrow f'(\pi/2)$ does not exist
$f'(x)$ changes sign from positive to negative at $x = 0$.

**11. (d) :** $2222 \equiv 3 \bmod 7$
$(2222)^3 \equiv 27 \bmod 7 \equiv -1 \bmod 7$
$(\because\ 27 \equiv -1 \bmod 7)$
$\Rightarrow (2222)^{5553} \equiv (-1)^{1851} \bmod 7 \equiv -1 \bmod 7$
$(2222)^2 \equiv 9 \bmod 7$
$(2222)^{5555} \equiv -9 \bmod 7 \equiv 5 \bmod 7$
$(\because\ -9 \equiv 5 \bmod 7)$

**12. (d) :** By Cosine rule

$$|z_1+z_2|=\sqrt{|z_1|^2+|z_2|^2+2|z_1||z_2|\cos 60^\circ}=\sqrt{19}$$

$$|z_1-z_2|=\sqrt{|z_1|^2+|z_2|^2-2|z_1||z_2|\cos 60^\circ}=\sqrt{7}$$

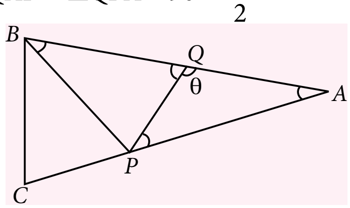
$$\left|\frac{z_1+z_2}{z_1-z_2}\right|=\sqrt{\frac{19}{7}}=\frac{\sqrt{133}}{7}$$

**13. (a) :** For $x=\dfrac{1}{f(y)} \Rightarrow f(1)=\dfrac{y^q}{(f(y))^p}$

$\Rightarrow f(y)=\dfrac{y^{q/p}}{(f(1))^{1/p}}$, $y = 1 \Rightarrow f(1) = 1 \Rightarrow f(y) = y^{q/p}$
$f(xy^{q/p}) = x^p y^q$
For $y = z^{p/q} \Rightarrow f(xz) = x^p z^p \Rightarrow f(x) = x^p$

$\dfrac{q}{p}=p \Rightarrow q=p^2$

**14. (c):** $\angle QAP=\angle QPA=90^\circ-\dfrac{\theta}{2}$




$\angle PQB = \angle PBQ = 180^\circ - \theta$
$\angle BCA=\angle ABC=\angle BPC=45^\circ+\dfrac{\theta}{4}$

Now $\left(90^\circ-\dfrac{\theta}{2}\right)+(2\theta-180^\circ)+\left(45^\circ+\dfrac{\theta}{4}\right)=180^\circ$

$\Rightarrow \theta=\dfrac{5\pi}{7} \Rightarrow 7\theta = 5\pi \Rightarrow 4\theta = 5\pi - 3\theta$
$\Rightarrow \tan 4\theta = -\tan 3\theta$

**15. (d) :** Let the circle will be $(x - h)^2 + (y - k)^2 = k^2$
$\Rightarrow (-1 - h)^2 + (1 - k)^2 = k^2$
$\Rightarrow h^2 + 2h + 2 - 2k = 0$
$\because h$ is real $D \ge 0 \Rightarrow k\ge\dfrac{1}{2}$.

**16. (a) :** Let $l, m, n$ be direction ratios of the normal
$\therefore\ l + m + n = 0$
and $l + m + nd = 0$
$\Rightarrow n(1 - d) = 0 \Rightarrow n = 0$
$\therefore$ Direction cosines are

$$\left(\frac{1}{\sqrt2},-\frac{1}{\sqrt2},0\right) \text{ or } \left(-\frac{1}{\sqrt2},\frac{1}{\sqrt2},0\right)$$

**17. (c):** $f(x)+f'''(3)=x^3+f'(1)x^2+f''(2)x$ ... (1)
Diff. successively w.r.t '$x$', we have
$f'(x) = 3x^2 + 2xf'(1) + f''(2)$ ... (2)
$f''(x) = 6x + 2f'(1)$ ... (3) $f'''(x) = 6$ ... (4)
Put $x = 1$ in (2) $x = 2$ in (3), $x = 3$ in (4)
$f'''(3) = 6$, $f'(1) = 3 + 2f'(1) + f''(2)$
$f''(2) = 12 + 2f'(1)$
Solving we get $f'''(3) = 6$, $f'(1) = -5$, $f''(2) = 2$

**18. (b) :** If length of medians be $m_a$, $m_b$, $m_c$ then
$2(m_a + m_b + m_c) < 3(a + b + c) < 4(m_a + m_b + m_c)$
$\Rightarrow 6 < a + b + c < 12 \Rightarrow s < 6$

Also, $[(s-a)(s-b)(s-c)]^{1/3}<\dfrac{s-a+s-b+s-c}{3}<2$
$\therefore\ (s - a)(s - b)(s - c) < 8$
$\therefore\ s(s - a)(s - b)(s - c) < 48 \Rightarrow \Delta<4\sqrt3$

**19. (b) :** Coordinate of any point on line may be taken as $(\alpha + r\cos\theta, \beta + r\sin\theta)$. If point lies on the curve then
$a(\alpha + r\cos\theta)^2 + 2h(\alpha + r\cos\theta)(\beta + r\sin\theta) + b(\beta + r\sin\theta)^2 = 1$

$$PQ\cdot PR=r_1\cdot r_2=\left|\frac{a\alpha^2+2h\alpha\beta+b\beta^2-1}{a\cos^2\theta+b\sin^2\theta+h\sin^2\theta}\right|$$

It will be independent of $\theta$ if $a = b$ and $h = 0$.

**20. (b) :** If it is column wise orthogonal then take

$$A^TA=\begin{bmatrix}1&1&1\\-2&1&1\\0&1&-1\end{bmatrix}\begin{bmatrix}1&-2&0\\1&1&1\\1&1&-1\end{bmatrix}=\begin{bmatrix}3&0&0\\0&6&0\\0&0&2\end{bmatrix}$$

Prefactor matrix will become inverse of $A$ if 1st row is divided by 3, 2nd by 6, 3rd by 2.

**21. (b) :** Using multinomial theorem,

$$\sum\frac{10!}{x_1!x_2!x_3!}2^{\frac{x_1}{2}}\cdot 3^{\frac{x_2}{3}}\cdot 6^{\frac{x_3}{6}}$$

where $x_1 + x_2 + x_3 = 10$

For rational terms possibilities are

$$x_1 = 4,\ x_2 = 6,\ x_3 = 0$$
$$x_1 = 10,\ x_2 = 0,\ x_3 = 0$$
$$x_1 = 4,\ x_2 = 0,\ x_3 = 6$$

So, three rational terms.

**22. (c):** Total number of ways of giving 10 gifts to 5 students so that each get atleast one and a particular student gets atleast 5 gifts.

$$= {}^{10}C_5 \frac{(2+1+1+1)!}{2!1!1!1!\times 3!}\times 4! + {}^{10}C_6 \frac{(1+1+1+1)!4!}{1!1!1!1!4!} = 65520$$

**23. (d) :** $\text{S.D} = \left|\dfrac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$

$$(\vec{a}_2-\vec{a}_1) = -4\hat{i}+16\hat{j}$$
$$(\vec{b}_1\times\vec{b}_2) = 16\hat{i}+16\hat{j}+16\hat{k}$$
$$\frac{(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|} = \frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$$

$$\text{S.D.} = \left|\frac{1}{\sqrt{3}}(-4+16)\right| = \frac{12}{\sqrt{3}} = 4\sqrt{3}$$

**24. (c):** $f(x) = x^7 + 14x^5 + 16x^3 + 30x - 560$

$\therefore\ f'(x) = 7x^6 + 70x^4 + 48x^2 + 30 > 0\ \forall\ x$

$\therefore\ f(x)$ is increasing $\forall\ x$

$\therefore$ It is odd degree polynomial.

It has only one real root.

**25. (d) :** $S = \dfrac{1}{n^4}\displaystyle\prod_{r=1}^{2n}(n^2+r^2)^{1/n}$

$$\log S = \sum_{r=1}^{2n}\frac{1}{n}\log(n^2+r^2) - 4\log n$$
$$= \frac{1}{n}\sum_{r=1}^{2n}\log\left(n^2\left(1+\left(\frac{r}{n}\right)^2\right)\right) - 4\log n$$
$$= \frac{1}{n}\left[\sum_{r=1}^{2n}2\log n + \sum_{r=1}^{2n}\log\left(1+\left(\frac{r}{n}\right)^2\right)\right] - 4\log(n)$$
$$= 4\log(n) + \frac{1}{n}\sum_{r=1}^{2n}\log\left(1+\left(\frac{r}{n}\right)^2\right) - 4\log(n)$$
$$\therefore\ \lim_{n\to\infty}\log S = \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n}\log\left(1+\left(\frac{r}{n}\right)^2\right)$$
$$= \int_0^2 \log(1+x^2)\,dx = \int_0^2 \log(x^2-4x+5)dx$$
$$\left(\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

**26. (b) :** $a, b$ are factors of the form $2^{a_1}5^{b_1}11^{c_1}, 2^{a_2}5^{b_2}11^{c_2}$ where $a_1, b_1, c_1, a_2, b_2, c_2$ are non negative integers.

Since LCM of $a$, $b$ is $2^3\ 5^7\ 11^{13}$,

$\max\{a_1, a_2\} = 3$, $\max\{b_1, b_2\} = 7$ and $\max(c_1, c_2) = 13$.

Hence $\{a_1, a_2\}$ can be (0, 3), (1, 3), (2, 3), (3, 3), (3, 2), (3, 1), (3, 0) (one of the number is 3 and other number can be any where from 0 to 3 ) giving us 7 choices. Similarly $(b_1, b_2)$ has 15 choices and $(c_1, c_2)$ has 27 choices.

Hence total number of choices = 7 × 15 × 27 = 2835.

**27. (b) :** Let $z = xy$

$$z^4 = (xy)^4 = \frac{1}{(ab)^2}(a^2x^4)(b^2y^4)$$

$z$ is maximum when $z^4$ is maximum.

*i.e.* when $a^2x^4$ and $b^2y^4$ are equal.

$$\frac{a^2x^4}{1} = \frac{b^2y^4}{1} = \frac{c^6}{2}$$

Maximum value of $z^4 = \dfrac{1}{(ab)^2}\left(\dfrac{c^6}{2}\right)\left(\dfrac{c^6}{2}\right)$

Maximum value of $z = \dfrac{c^3}{\sqrt{2ab}}$

**28. (b) :** Let $g(x) = \int_0^x (\sin x + x\cos x)\,dx$

$g(0) = 0 = g(\pi)$

By Rolle's Theorem at least one root $\in (0, \pi)$

For $g'(x) = 0$

*i.e.* $\sin x + x\cos x = 0$.

**29. (d) :** Equation of normal from $(h, k)$ is

$am^3 + (2a - h)\,m - k = 0$

$$\Sigma t_1 = 0,\ \Sigma t_1t_2 = \frac{2a-h}{a}, \Pi t_1 = \frac{k}{a}$$

The vertices of the triangle formed by the tangents are $A(at_1t_2, a(t_1 + t_2))$, $B(at_2t_3, a(t_2 + t_3))$, $C(at_3t_1, a(t_3 + t_1))$

Centroid $G = \left(\dfrac{a}{3}\Sigma t_1t_2, \dfrac{2a}{3}\Sigma t_1\right)$

$$= \left(\frac{2a-h}{3}, 0\right)$$

**30. (a) :** Let $z = \pi\cos\theta$

$$z + \frac{1}{z} = \sqrt{3} \Rightarrow \left(\pi + \frac{1}{\pi}\right)\cos\theta = \sqrt{3}$$
$$\left(\pi - \frac{1}{\pi}\right)\sin\theta = 0 \Rightarrow \theta \neq 0 \text{ and } \pi = 1$$
$$\Rightarrow \theta = \frac{\pi}{6} \Rightarrow \left(z^n + \frac{1}{z^n}\right)^2 = 4\cos^2\frac{\pi r}{6}$$
$$\Sigma\left(z^n + \frac{1}{z^n}\right)^n = 4\left(\cos^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} + \cos^2\frac{\pi}{2} + \cos^2\frac{2\pi}{3} + \cos^2\frac{5\pi}{6}\right)$$
$$= 8$$

**31. (c):** $f(b)f(c) < 0, f(c)f(d) < 0$
$f(x) = 0$ has atleast 4 roots $a, c_1, c_2, e$
[by intermediate value property, $f(x) = 0$ has at least one root $c_1$ in $(b, c)$]
And at least one root $c_2$ in $(c, d)$
By Rolle's theorem $f'(x)$ has 3 roots in $(a, c_1)$, $(c_1, c_2)$, $(c_2, e)$
$\therefore\ f(x)f'(x) = 0$ has at least $4 + 3 = 7$ roots

$\Rightarrow\ g(x) = \dfrac{d}{dx}(f(x)f'(x)) = 0$ has at least 6 roots

**32. (a) :** Given :
$(\sin A - \cos A)(\sin B - \cos B)(\sin C - \cos C) = 1$
$\Rightarrow (\tan A - 1)(\tan B - 1)(\tan C - 1) = \sec A \sec B \sec C$
$\Rightarrow \left(\sum \tan A - \sum \tan A \tan B + \sum \tan A - 1\right) = \pi \sec A$
$\Rightarrow 2\sum \tan A - \sum \tan A \tan B = 1 + \pi \sec A$ ... (i)
But in a $\Delta ABC$, $\sum \tan A \tan B = 1 + \pi \sec A$ ... (ii)
$\Rightarrow 2\sum \tan A = 2\sum \tan A \tan B$
$\Rightarrow \sum \tan A \tan B = \sum \tan A$

**33. (c):** If $\alpha = \displaystyle\int_0^{\pi/2} \frac{\sin x}{1 + \sin x + \cos x}dx$ then

$$\alpha = \int_0^{\pi/2} \frac{\cos x}{1 + \sin x + \cos x}dx$$

$$\therefore\ \log 2 + 2\alpha = \int_0^{\pi/2} dx = \frac{\pi}{2}$$

**34. (c):** $\displaystyle\lim_{x\to\infty} \frac{4x^2 + 3x + 5}{x + 1 + x^{k-1}}$ exist when in denominator power of $x$ is more.
*i.e.*, $k - 1 \geq 2 \ \Rightarrow\ k \geq 3$.

**35. (c):** $P = (a\cos\theta, b\sin\theta)$

The tangent at $P$ is $\dfrac{x}{a}\cos\theta + \dfrac{y}{b}\sin\theta = 1$

$$OA = \frac{ab}{\sqrt{a^2\sin^2\theta + b^2\cos^2\theta}}$$

Normal at $P$ is $\dfrac{ax}{\cos\theta} - \dfrac{by}{\sin\theta} = (a^2 - b^2)$

$$B = \left(\frac{(a^2 - b^2)}{a}\cos\theta, 0\right)$$

$$(PB)^2 = b^2\sin^2\theta + \left(a\cos\theta - \frac{(a^2 - b^2)\cos\theta}{a}\right)^2$$

$$PB = \frac{b\sqrt{a^2\sin^2\theta + b^2\cos^2\theta}}{a}$$

$(OA)\cdot(PB) = b^2$

**36. (b) :** Given parabola can be rewritten as
$(y - 1)^2 = 4(x - 2)$
Equation of any normal to the given parabola is
$y - 1 = m(x - 2) - 2m - m^3$
$\Rightarrow\ y = mx - 4m - m^3 + 1$
Since, this passes through $(2a, 0)$
$\therefore\ m^3 + 2m(2 - a) - 1 = 0$ will have three distinct and real values of $m$.
Also, $3m^2 + 2(2 - a) = 0$ will have all real and distinct root $\Rightarrow\ a > 2$

**37. (b) :** Using $C_1 \to C_1 - pC_2 - C_3$

$$\begin{vmatrix} 0 & x & y \\ 0 & y & z \\ -(p^2x + py) - (py + z) & px + y & py + z \end{vmatrix}$$

$\Rightarrow\ (y^2 - xz)(p^2x + py + py + z) = 0$
$x, y, z$ are in G.P.

**38. (d) :** $x\displaystyle\int_0^x g^2(t)dt = 2\left(\int_0^x g(t)dt\right)^2$ ... (1)

$$\left(\int_0^x g(t)dt = \int_0^x g(x - t)dt\right)$$

Differentiating, $\displaystyle\int_0^x g^2(t)dt + xg^2(x) = 4g(x)\int_0^x g(t)dt$

$\Rightarrow\ x\displaystyle\int_0^x g^2(t)dt + x^2g^2(x) = 4xg(x)\int_0^x g(t)dt$ ... (2)

From (1) & (2),

$$2\left(\int_0^x g(t)dt\right)^2 - 4xg(x)\int_0^x g(t)dt + x^2g^2(x) = 0$$

$$\Rightarrow\ \int_0^x g(t)dt = \frac{2 \pm \sqrt{2}}{2}xg(x)$$

Differentiating, $\dfrac{g'(x)}{g(x)} = \dfrac{1 \pm \sqrt{2}}{x}$

$g(x) = c \cdot x^{1 \pm \sqrt{2}}$

$g(1) = 1 \ \Rightarrow\ c = 1$

**39. (d) :** $f(x) = \cos^2 x + \cos^2 2x + \cos^2 3x$
$= 1 + \cos^2 x + \cos^2 2x - \sin^2 3x$
$= 1 + \cos^2 x + \cos 5x \cdot \cos x$
$= 1 + \cos x(\cos x + \cos 5x)$
$= \cos x \cos 2x \cos 3x = 0.$

**40. (b) :** Any point on given ellipse is $(4a\cos\theta, 2a\sin\theta)$,
slope of tangent at which is $-\dfrac{1}{2}\dfrac{\cos\theta}{\sin\theta}$.
This is a tangent to circle with centre at (1, 0) if

$$-\frac{1}{2}\frac{\cos\theta}{\sin\theta} \times \frac{2\sin\theta}{4\cos\theta - 1} = -1 \ \Rightarrow\ \cos\theta = \frac{1}{3}$$

$\therefore$ Radius of the required circle is

$$a\sqrt{(4\cos\theta-1)^2+4\sin^2\theta}$$

$$=a\sqrt{\left(\frac{4}{3}-1\right)^2+4\left(1-\frac{1}{9}\right)}=a\sqrt{\frac{11}{3}}$$

**41. (a)**

**42. (c):** The 3 vertices can be chosen among 8 is $^8C_3$ = 56 ways. For an equilateral triangle to be formed, sides must be chosen along face diagonals. Since through each vertex, 3 face diagonals can be drawn, totally $\frac{8\times 3}{3}=8$ (each equilateral triangle is counted 3 times here ) such triangles .

$\therefore$ Required Probability = $\frac{8}{56}=\frac{1}{7}$

**43. (a) :** $10^{99} = 2^{99}\cdot 5^{99}$ has totally $(100)^2$ divisors of the form $2^a\cdot 5^b$ $(0 \le a, b \le 99)$. Of these multiples of $10^{88}$ are of the form $2^a\cdot 5^b$ $(88 \le a, b \le 99)$. So there are $12\cdot 12 = 12^2$ choices for both $a$ and $b$.

Hence, the probability required is $\frac{12^2}{100^2}=\left(\frac{3}{25}\right)^2$.

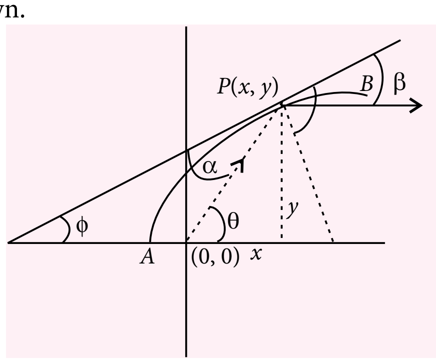
**44. (c):** For an even license plate, there must be $2K$ zeros $(0 \le K \le 4)$ and 8-2$K$ non zero digits, each of which has 9 choices. There are $^8C_{2K}$ ways to choose $2K$ places for 0's and $(^8C_{2K})9^{8-2K}$ plates have exactly $2K$ zeroes.

Therefore the required answer is

$$\sum_{K=0}^{4}(^8C_{2K})\,9^{8-2K}={}^8C_0\,9^8+{}^8C_2 9^6+{}^8C_4 9^4+{}^8C_6 9^2+{}^8C_8$$

$$=\frac{(9+1)^8+(9-1)^8}{2}=\frac{10^8+8^8}{2}$$

**45. (b) :** Let the shape of surface of a curved mirror be as shown.




By law of reflection, $\alpha = \beta$, but $\phi = \beta$

Also $\theta = \alpha + \phi = 2\beta = 2\phi$

Since $\tan\theta=\frac{y}{x}$, we get

$$\frac{y}{x}=\tan\theta=\tan 2\phi=\frac{2\tan\phi}{1-\tan^2\phi}=\frac{2\frac{dy}{dx}}{1-\left(\frac{dy}{dx}\right)^2}$$

Solving gives

$$\frac{dy}{dx}=\frac{-x\pm\sqrt{x^2+y^2}}{y}\quad\text{or}\quad\frac{xdx+ydy}{\sqrt{x^2+y^2}}=\pm dx$$

$$\Rightarrow\ \pm\sqrt{x^2+y^2}=x+c$$

squaring gives $y^2 = 2cx + c^2$ which is a parabola.

**46. (d) :** Area = $2\int_{-2}^{0}ydx=2\int_{-2}^{0}x^2\sqrt{x+2}\,dx$

$$=4\int_{0}^{\sqrt{2}}(z^2-2)^2z^2dz\ \ (\text{where }\sqrt{x+2}=z)$$

$$=4\left[\frac{z^7}{7}-\frac{4z^5}{5}+\frac{4z^3}{3}\right]_0^{\sqrt{2}}=\frac{256\sqrt{2}}{105}$$

**47. (a) :** $37^{n+2} = (5 \times 7 + 2)^{n+2}$

= a multiple of 7 + $2^{n+2}$

$16^{n+1} = (2 \times 7 + 2)^{n+1}$

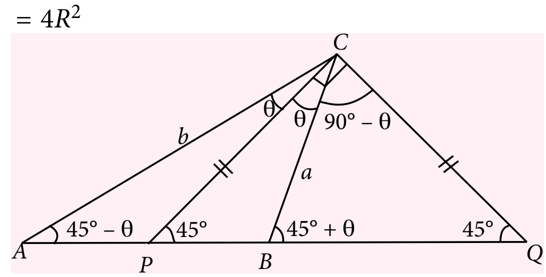
= a multiple of 7 + $2^{n+1}$

and $30^n = (4 \times 7 + 2)^n$ = a multiple of 7 + $2^n$

**48. (c):** $a^2 + b^2 = 4R^2(\sin^2(45° - \theta) + \sin^2(135° - \theta)$

$= 4R^2(\sin^2(45° - \theta) + \cos^2(45° - \theta))$

$= 4R^2$




**49. (b) :** Solving $y^2 = 4ax$, $x^2 + y^2 - 6ax + a^2 = 0$, we get

$\therefore$ Points of contact are $(a, 2a)$, $(a, -2a)$

Hence tangents intersect at $(-a, 0)$

**50. (d) :** $f(x) \ge 0\ \forall\ x \ge 0$ .... (1)

$f'(x)\cos x - f(x)\sin x \le 0 \Rightarrow (f(x)\cos x)' \le 0$

Let $G(x) = f(x)\cos x$ is a decreasing function.

$$\Rightarrow\ G\left(\frac{\pi}{2}\right)\ge G\left(\frac{5\pi}{3}\right)\ \Rightarrow\ G\left(\frac{5\pi}{3}\right)\le 0$$

$$\Rightarrow\ f\left(\frac{5\pi}{3}\right)\le 0 \qquad \text{.... (2)}$$

From (1) and (2), $f\left(\frac{5\pi}{3}\right)=0$

# ACE YOUR WAY CBSE XII

# PRACTICE PAPER 2016

Time Allowed : 3 hours | Maximum Marks : 100

**GENERAL INSTRUCTIONS**

(i) All questions are compulsory.
(ii) Please check that this Question Paper contains 26 Questions.
(iii) Marks for each question are indicated against it.
(iv) Questions 1 to 6 in Section-A are Very Short Answer Type Questions carrying **one mark** each.
(v) Questions 7 to 19 in Section-B are Long Answer I Type Questions carrying **4 marks** each.
(vi) Questions 20 to 26 in Section-C are Long Answer II Type Questions carrying **6 marks** each.
(vii) Please write down the serial number of the Question before attempting it.

## SECTION-A

**1.** Evaluate : $\int_0^{1.5}[x]dx$ (where $[x]$ is greatest integer function).

**2.** If $\begin{bmatrix} x+3y & y \\ 7-x & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$, find the values of $x$ and $y$.

**3.** Find the value of $p$, for which $\vec{a}=3\hat{i}+2\hat{j}+9\hat{k}$ and $\vec{b}=\hat{i}+p\hat{j}+3\hat{k}$ are parallel vectors.

**4.** Find the integrating factor of the linear differential equation $\frac{dx}{dy}+(\tan y)x=\sec^2 y$

**5.** Evaluate : $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx.$

**6.** If the equation of a line $AB$ is $\frac{x-3}{1}=\frac{y+2}{-2}=\frac{z-5}{4}$, find the direction ratios of a line parallel to $AB$.

## SECTION-B

**7.** Evaluate : $\int x\sin^{-1}x\,dx$

**8.** If $y = 3\cos(\log x) + 4\sin(\log x)$, then show that $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0.$

**9.** Solve the differential equation :

$$\frac{dy}{dx}+y\sec^2 x=\tan x\sec^2 x; y(0)=1$$

**OR**

Solve the following differential equation :

$$x\frac{dy}{dx}+y=x\log x;\ x\neq 0$$

**10.** Find the equation of the line passing through the point $P(4, 6, 2)$ and the point of intersection of the line $\frac{x-1}{3}=\frac{y}{2}=\frac{z+1}{7}$ and the plane $x+y-z=8$.

**11.** By using properties of determinants, prove the following :

$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}=(5x+4)(4-x)^2$$

**12.** Prove that $\int_1^3 \frac{dx}{x^2(x+1)}=\frac{2}{3}+\log\frac{2}{3}$.

**13.** Find a vector of magnitude 5 units perpendicular to each of the vectors $(\hat{a}+\hat{b})$ and $(\hat{a}-\hat{b})$ where $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$.

**OR**

The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of $\lambda$.

**14.** Prove the following :

$\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)=\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$.

**15.** Let $X$ be a non-empty set and * be a binary operation on $P(X)$ (the power set of set $X$) defined by

$$A * B = A \cup B \text{ for all } A, B \in P(X)$$

Prove that '*' is both commutative and associative on $P(X)$. Find the identity element with respect to '*' on $P(X)$. Also, show that $\phi \in P(X)$ is the only invertible element of $P(X)$.

**OR**

Show that the relation $R$ in the set of real numbers, defined as $R = \{(a, b) : a \le b^2\}$ is neither reflexive, nor symmetric, nor transitive.

**16.** Find the inverse of matrix $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$, if exists, using elementary row transformation.

**17.** If $(\cos x)^y = (\sin y)^x$, find $\frac{dy}{dx}$.

**18.** The probability that student entering a university will graduate is 0.4. Find the probability that out of 3 students of the university:

(i) none will graduate,

(ii) only one will graduate,

(iii) all will graduate.

**OR**

A football match may be either won, drawn or lost by the host country's team. So there are three ways of forecasting the result of any one match, one correct and two incorrect. Find the probability of forecasting at least three correct results for four matches.

**19.** If $f(x) = \begin{cases} \frac{x-5}{|x-5|}+a, & \text{if } x<5 \\ a+b, & \text{if } x=5 \\ \frac{x-5}{|x-5|}+b, & \text{if } x>5 \end{cases}$ is a continuous function. Find the value of $a$ and $b$.

## SECTION-C

**20.** Find the probability distribution of the number of white balls drawn in a random draw of 3 balls without replacement from a bag containing 4 white and 6 red balls. Also find the mean and variance of the distribution.

**21.** Every gram of wheat provides 0.1 gm of proteins and 0.25 gm of carbohydrates. The corresponding values for rice are 0.05 gm and 0.5 gm respectively. Wheat costs ₹ 4 per kg and rice ₹ 6 per kg. The minimum daily requirements of proteins and carbohydrates for an average child are 50 gms and 200 gms respectively. In what quantities should wheat and rice be mixed in the daily diet to provide minimum daily requirement of proteins and carbohydrates at minimum cost. Frame a L.P.P. and solve it graphically. What are the two uses of proteins?

**22.** Find the equation of the plane passing through the points (3, 4, 1) and (0, 1, 0) and parallel to the line $\frac{x+3}{2}=\frac{y-3}{7}=\frac{z-2}{5}$

**OR**

Find the distance of the point (–2, 3, –4) from the line $\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$

**23.** A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m$^3$. If building of tank costs ₹ 70 per sq. metre for the base and ₹ 45 per sq. metre for sides, what is the cost of least expensive tank?

**OR**

Show that the height of the cone of maximum volume that can be inscribed in a sphere of radius 12 cm is 16 cm.

**24.** Solve : $x\frac{dy}{dx} + y - x + xy \cot x = 0 \; (x \neq 0)$

**25.** Find the ratio of shaded to unshaded region into which curve $y^2 = 6x$ divides the region bounded by $x^2 + y^2 = 16$.

**26.** Consider $f: R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that $f$ is invertible. Find the inverse of $f$.

## SOLUTIONS

**1.** $\int_0^{1.5}[x]dx = \int_0^1 0dx + \int_1^{1.5} 1dx = 0 + [x]_1^{1.5}$
$= 1.5 - 1 = 0.5$

**2.** Given $\begin{bmatrix} x+3y & y \\ 7-x & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$

$y = -1$ and $7 - x = 0$

$\Rightarrow \quad x = 7, y = -1$

**3.** Since $\vec{a} \| \vec{b}$, therefore $\vec{a} = \lambda\vec{b}$

$\Rightarrow \quad 3\vec{i} + 2\vec{j} + 9\vec{k} = \lambda(\vec{i} + p\vec{j} + 3\vec{k})$

$\Rightarrow \quad \lambda = 3, 2 = \lambda p, 9 = 3\lambda$

or $\quad \lambda = 3, p = \dfrac{2}{3}$

**4.** I.F. $= e^{\int \tan y\, dy} = e^{\log|\sec y|} = \sec y$

**5.** Let $I = \int \dfrac{\cos\sqrt{x}}{\sqrt{x}}dx \quad$ Let $\sqrt{x} = t \Rightarrow \dfrac{1}{2\sqrt{x}}dx = dt$

$\Rightarrow \quad I = \int \cos t \,.\, 2dt \Rightarrow I = 2\sin t + C$

$\Rightarrow \quad I = 2\sin\sqrt{x} + C$

**6.** The direction ratios of line parallel to $AB$ is $1, -2$ and $4$.

**7.** Let $I = \int \underset{\text{II}}{x} \underset{\text{I}}{\sin^{-1}x}\, dx$

$$I = \sin^{-1}x.\frac{x^2}{2} - \int \frac{x^2}{2\sqrt{1-x^2}}dx$$

$$\Rightarrow \quad I = \frac{x^2}{2}\sin^{-1}x + \frac{1}{2}\int \frac{1-x^2-1}{\sqrt{1-x^2}}dx$$

$$= \frac{x^2}{2}\sin^{-1}x + \frac{1}{2}\int\sqrt{1-x^2}\,dx - \frac{1}{2}\sin^{-1}x$$

$$= \frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\sin^{-1}x + \frac{1}{2}\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right] + C$$

$$= \frac{x^2}{2}\sin^{-1}x - \frac{1}{4}\sin^{-1}x + \frac{1}{4}x\sqrt{1-x^2} + C$$

$$= \frac{1}{4}\left[(2x^2-1)\sin^{-1}x + x\sqrt{1-x^2}\right] + C$$

**8.** We have $y = 3\cos(\log x) + 4\sin(\log x)$ ...(i)

Differentiating (i) w.r.t.$x$, we get

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-3\sin(\log x)}{x} + \frac{4\cos(\log x)}{x}$$

$$\Rightarrow \quad x\frac{dy}{dx} = -3\sin(\log x) + 4\cos(\log x) \quad \text{...(ii)}$$

Differentiating again (ii) w.r.t.$x$, we get

$$\Rightarrow \quad x\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{-3\cos(\log x)}{x} - \frac{4\sin(\log x)}{x}$$

$$\Rightarrow \quad x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -y \Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

**9.** Given differential equation, is

$$\frac{dy}{dx} + y\sec^2 x = \tan x \sec^2 x;\ y(0) = 1$$

It is linear form of the type $\dfrac{dy}{dx} + Py = Q$

Here, $P = \sec^2 x$ and $Q = \tan x \sec^2 x$

$\therefore \quad$ I.F. $= e^{\int P dx} = e^{\int \sec^2 x dx} = e^{\tan x}$

Solution is given by

$y \times$ I.F. $= \int Q \times$ I.F. $dx$

$ye^{\tan x} = \int \tan x \sec^2 x\, e^{\tan x} dx$

Let $\quad e^{\tan x} = t \Rightarrow \tan x = \log t$

$\Rightarrow \quad e^{\tan x}\sec^2 x\, dx = dt$

$\therefore \quad y\, e^{\tan x} = \int \log t\, dt = t\log t - t \quad$ (Integrating by parts)

$\Rightarrow \quad y\, e^{\tan x} = e^{\tan x}\cdot \tan x - e^{\tan x} + C \quad$ ...(i)

Now, from given value $x = 0$, $y = 1$, we get

$1 \cdot e^0 = e^0 \cdot 0 - e^0 + C \Rightarrow C = 2$

Putting the value of $C$ in (i), we get

$y\, e^{\tan x} = e^{\tan x}\cdot \tan x - e^{\tan x} + 2.$

$\Rightarrow \quad y = \tan x - 1 + 2e^{-\tan x}$

**OR**

Given, $x\dfrac{dy}{dx} + y = x\log x \Rightarrow \dfrac{dy}{dx} + \dfrac{y}{x} = \log x$

This is linear differential equation of the type $\dfrac{dy}{dx} + Py = Q$

Here $P = \dfrac{1}{x}, Q = \log x$

Integrating factor I.F. $= e^{\int \frac{1}{x}dx} = e^{\log x} = x$

Soution is given by

$y \times$ I.F. $= \int Q \times$ I.F. $dx$

$y \cdot x = \int x \cdot \log x\, dx$

$$\Rightarrow \quad xy = \log x.\frac{x^2}{2} - \int \frac{x^2}{2}.\frac{1}{x}dx$$

$$\Rightarrow \quad xy = \frac{x^2\log x}{2} - \frac{1}{2}\frac{x^2}{2} + C$$

$$\Rightarrow \quad y = \frac{x^2}{2}\left(\log x - \frac{1}{2}\right) + C$$

**10.** Let $\frac{x-1}{3}=\frac{y}{2}=\frac{z+1}{7}=\lambda$ ...(i)

Coordinates of any general point on line (i) is of the form $\equiv(1+3\lambda, 2\lambda, -1+7\lambda)$

For point of intersection

$(1+3\lambda)+2\lambda-(7\lambda-1)=8$

$\Rightarrow$ $1+3\lambda+2\lambda-7\lambda+1=8$

$\Rightarrow$ $-2\lambda=6 \Rightarrow \lambda=-3$

Point of intersection $\equiv(-8,-6,-22)$

$\therefore$ Required equation of line passing through $P(4,6,2)$ and $Q(-8,-6,-22)$ is

$$\frac{x-4}{4+8}=\frac{y-6}{6+6}=\frac{z-2}{2+22}$$

$\therefore$ $\frac{x-4}{12}=\frac{y-6}{12}=\frac{z-2}{24}$. or $x-4=y-6=\frac{z-2}{2}$

**11.** L.H.S. $=\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$

Applying $C_1\to C_1+C_2+C_3$, we get

$$=\begin{vmatrix} 5x+4 & 2x & 2x \\ 5x+4 & x+4 & 2x \\ 5x+4 & 2x & x+4 \end{vmatrix}$$

Taking $5x+4$ common from $C_1$, we get

$$=(5x+4)\begin{vmatrix} 1 & 2x & 2x \\ 1 & x+4 & 2x \\ 1 & 2x & x+4 \end{vmatrix}$$

Applying $R_2\to R_2-R_1$; $R_3\to R_3-R_1$, we get

$$=(5x+4)\begin{vmatrix} 1 & 2x & 2x \\ 0 & 4-x & 0 \\ 0 & 0 & 4-x \end{vmatrix}$$

Expanding along $C_1$, we get

$=(5x+4)(4-x)^2=$ R.H.S.

**12.** Let L.H.S. $=I=\int_1^3 \frac{dx}{x^2(x+1)}$

Using partial fraction, we have

$$\frac{1}{x^2(x+1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+1}$$

$\Rightarrow$ $\frac{1}{x^2(x+1)}=\frac{Ax(x+1)+B(x+1)+Cx^2}{x^2(x+1)}$

$\Rightarrow$ $1=(A+C)x^2+(A+B)x+B$

Equating coefficient of similar terms, we have

$A+C=0$, $A+B=0$, $B=1$

On solving these three equations, we have

$A=-1$, $B=1$ and $C=1$

$\therefore$ $I=\int_1^3\frac{dx}{x^2(x+1)}=\int_1^3\left(-\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x+1}\right)dx$

$=\left[-\log|x|-\frac{1}{x}+\log|x+1|\right]_1^3$

$=-\log 3-\frac{1}{3}+\log 4+0+1-\log 2$

$=\frac{2}{3}+(2\log 2-\log 2-\log 3)$ $(\because \log 4=\log 2^2)$

$=\frac{2}{3}+(\log 2-\log 3)=\frac{2}{3}+\log\frac{2}{3}=$ R.H.S.

**13.** We have, $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$

$\therefore$ $\vec{a}+\vec{b}=2\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{a}-\vec{b}=-\hat{j}-2\hat{k}$

Thus, $(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix}$

$=\hat{i}(-6+4)-\hat{j}(-4-0)+\hat{k}(-2-0)$

$=-2\hat{i}+4\hat{j}-2\hat{k}$

$\therefore$ $|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})|=\sqrt{(-2)^2+(4)^2+(-2)^2}$

$=\sqrt{4+16+4}=\sqrt{24}=2\sqrt{6}$

$\therefore$ Required vector $=5\left\{\frac{(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})}{|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})|}\right\}$

$=\frac{5(-2\hat{i}+4\hat{j}-2\hat{k})}{2\sqrt{6}}=\frac{-5}{\sqrt{6}}\hat{i}+\frac{10}{\sqrt{6}}\hat{j}-\frac{5}{\sqrt{6}}\hat{k}$

**OR**

Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ and $\vec{c}=\lambda\hat{i}+2\hat{j}+3\hat{k}$

Thus, $\vec{b}+\vec{c}=(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}$

$\therefore$ $|\vec{b}+\vec{c}|=\sqrt{(2+\lambda)^2+(6)^2+(-2)^2}$

$=\sqrt{4+\lambda^2+4\lambda+36+4}=\sqrt{\lambda^2+4\lambda+44}$

Now, According to question, $\vec{a}\cdot\frac{\vec{b}+\vec{c}}{|\vec{b}+\vec{c}|}=1$

$\Rightarrow$ $(\hat{i}+\hat{j}+\hat{k})\cdot\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}=1$

$\Rightarrow$ $\frac{(2+\lambda)+6-2}{\sqrt{\lambda^2+4\lambda+44}}=1 \Rightarrow \lambda+6=\sqrt{\lambda^2+4\lambda+44}$

Squaring both sides, we have

$\lambda^2+36+12\lambda=\lambda^2+4\lambda+44 \Rightarrow 8\lambda=8 \Rightarrow \lambda=1$

**14.** L.H.S. $= \tan^{-1}\frac{1}{4} + \tan^{-1}\left(\frac{2}{9}\right)$

$$= \tan^{-1}\left(\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\cdot\frac{2}{9}}\right) = \tan^{-1}\left(\frac{\frac{17}{36}}{\frac{34}{36}}\right)$$

$$= \tan^{-1}\frac{1}{2} = \frac{1}{2}\left(2\tan^{-1}\frac{1}{2}\right)$$

$$= \frac{1}{2}\cos^{-1}\left(\frac{1-\frac{1}{4}}{1+\frac{1}{4}}\right) \quad \left[\because 2\tan^{-1}A = \cos^{-1}\frac{1-A^2}{1+A^2}\right]$$

$$= \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right) = \text{R.H.S.}$$

**15.** For any $A, B, C \in P(X)$, we have

$A \cup B = B \cup A$ and $(A \cup B) \cup C = A \cup (B \cup C)$

$\Rightarrow \quad A * B = B * A$ and $(A * B) * C = A * (B * C)$

Thus, '*' is both commutative and associative on $P(X)$.

Now, $A \cup \phi = A = \phi \cup A$ for all $A \in P(X)$

$\Rightarrow \quad A * \phi = A = \phi * A$ for all $A \in P(X)$.

So, $\phi$ is the identity element for '*' on $P(X)$.

Let $A \in P(X)$ be an invertible element. Then, there exists $S \in P(X)$ such that

$A * S = \phi = S * A \quad \Rightarrow \quad A \cup S = \phi = S \cup A$

$\Rightarrow \quad S = \phi = A.$

Hence, $\phi$ is the only invertible element of $P(X)$.

**OR**

Given relation is $R = \{(a, b): a \le b^2\}$

*Reflexivity:*

Let $a \in$ real numbers.

$aRa \Rightarrow a \le a^2$

but if $a < 1$.

For example $a = \frac{1}{2} \quad \Rightarrow a^2 = \frac{1}{4} \quad a \not\le a^2$

Hence $R$ is not reflexive.

*Symmetry:*

Let $a, b \in$ real numbers

$aRb \Rightarrow a \le b^2$

But then $b \le a^2$ is not true

$\therefore \quad aRb \ne bRa$

For example, $a = 2, b = 5$

then $2 \le 5^2$ but $5 \le 2^2$ is not true.

Hence $R$ is not symmetric.

*Transitivity:*

Let $a, b, c \in$ real numbers

$aRb \Rightarrow \; a \le b^2$ and $bRc \Rightarrow b \le c^2$

Considering $aRb$ and $bRc$

$\Rightarrow \quad a \le c^4 \not\Rightarrow aRc$

For example, if $a = 2, b = -3, c = 1$

$aRb \Rightarrow 2 \le 9$

$bRc \Rightarrow -3 \le 1$

$aRc \Rightarrow 2 \le 1$ is not true.

Hence $R$ is not transitive

**16.** Consider $A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

We write $A = IA$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

[By performing $R_2 \to R_2 + 3R_1$]

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

[By performing $R_3 \to R_3 - 2R_1$]

$$\Rightarrow \begin{bmatrix} 1 & 0 & 10 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \text{A}$$

[By performing $R_1 \to R_1 + 3R_3$]

$$\Rightarrow \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & -\frac{11}{9} \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

[By performing $R_2 \to \frac{1}{9} R_2$]

$$\Rightarrow \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & -\frac{11}{9} \\ 0 & 0 & \frac{25}{9} \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ -\frac{5}{3} & \frac{1}{9} & 1 \end{bmatrix} A$$

[By performing $R_3 \to R_3 + R_2$]

$$\Rightarrow \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & -\frac{11}{9} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

[By performing $R_3 \to \frac{9}{25} R_3$]

$$\Rightarrow \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

[By performing $R_2 \to R_2 + \frac{11}{9} R_3$]

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

[By performing $R_1 \to R_1 - 10R_3$]

Hence, we obtain

$I = BA$

$\Rightarrow$ $B$ is inverse of $A$.

$$\text{Hence } A^{-1} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} = \frac{1}{25}\begin{bmatrix} 25 & -10 & -15 \\ -10 & 4 & 11 \\ -15 & 1 & 9 \end{bmatrix}$$

**17.** Given, $(\cos x)^y = (\sin y)^x$

Taking log on both sides, we get

$\therefore$ $\log (\cos x)^y = \log(\sin y)^x$

$\Rightarrow$ $y \log(\cos x) = x \log(\sin y)$

Differentiating both sides w.r.t.$x$, we get

$$y \frac{1}{\cos x} \cdot \frac{d}{dx} (\cos x) + \log(\cos x) \cdot \frac{dy}{dx} = x \cdot \frac{1}{\sin y} \cdot \frac{d}{dx} (\sin y) + \log \sin y \cdot 1$$

$$\Rightarrow -y \frac{\sin x}{\cos x} + \log(\cos x) \cdot \frac{dy}{dx} = x \frac{\cos y}{\sin y} \frac{dy}{dx} + \log \sin y$$

$$\Rightarrow -y \tan x + \log (\cos x) \frac{dy}{dx} = x \cot y \frac{dy}{dx} + \log \sin y$$

$$\Rightarrow \log (\cos x) \cdot \frac{dy}{dx} - x \cot y \frac{dy}{dx} = \log (\sin y) + y \tan x$$

$$\Rightarrow \frac{dy}{dx} [\log (\cos x) - x \cot y] = \log (\sin y) + y \tan x$$

$$\therefore \frac{dy}{dx} = \frac{\log(\sin y) + y \tan x}{\log(\cos x) - x \cot y}$$

**18.** Let $X$ denote the number of students who graduated.

Now, the probability that a student graduates, $p = 0.4$

$\therefore$ $q = 1 - p = 1 - 0.4 = 0.6$

(i) $P$ (none will graduate)

$= P(X = 0) = {}^nC_0 p^0 q^{n-0}$

$= {}^3C_0 (0.4)^0 \cdot (0.6)^3 = (0.6)^3 = 0.216$

(ii) $P$ (only one will graduate)

$= P(X = 1) = {}^nC_1 p^1 q^{n-1}$

$= {}^3C_1 (0.4)^1 \cdot (0.6)^2$

$= 3 \times (0.4) \times (0.6)^2 = 0.432$

(iii) $P$ (all will graduate)

$= P(X = 3) = {}^nC_3 . p^3 . q^{n-3}$

$= {}^3C_3 (0.4)^3 \cdot (0.6)^0$

$= (0.4)^3 = 0.064$

**OR**

We have,

The probability of correct result $= \frac{1}{3}$

and probability of incorrect result $= \frac{2}{3}$

$\therefore$ Probability of forecasting at least three correct results for four matches

= Probability of 3 correct and one incorrect or all correct

$$= 4 \times \left(\frac{1}{3}\right)^3 \times \frac{2}{3} + \left(\frac{1}{3}\right)^4$$

$$= \left(\frac{1}{3}\right)^4 (8+1) = \left(\frac{1}{3}\right)^4 \times 3^2 = \frac{1}{3^2} = \frac{1}{9}$$

**19.** We have $f(x) = \begin{cases} \dfrac{x-5}{|x-5|} + a, & \text{if } x < 5 \\ a + b, & \text{if } x = 5 \\ \dfrac{x-5}{|x-5|} + b, & \text{if } x > 5 \end{cases}$

Since $f$ is continuous, it must be continuous at $x = 5$

L.H.L. $= \lim\limits_{x \to 5^-} f(x) = \lim\limits_{x \to 5^-} \dfrac{x-5}{-(x-5)} + a = -1 + a$

R.H.L. $= \lim\limits_{x \to 5^+} f(x) = \lim\limits_{x \to 5^+} \dfrac{x-5}{(x-5)} + b = 1 + b$

and $f(5) = a + b$

Since, $f(x)$ is continuous at $x = 5$

$\therefore$ L.H.L. = R.H.L. = $f(5)$

$-1 + a = 1 + b = a + b$

$\Rightarrow$ $-1 + a = a + b$ $\Rightarrow$ $b = -1$

and $1 + b = a + b$ $\Rightarrow$ $a = 1$

Hence, $a = 1, b = -1$

**20.** Let $X$ be the number of white balls.

Therefore, we have

When $X = 0$, $p_1 = \dfrac{^6C_3}{^{10}C_3} = \dfrac{1}{6}$

When $X = 1$, $p_2 = \dfrac{^4C_1 \times {^6C_2}}{^{10}C_3} = \dfrac{1}{2}$

When $X = 2$, $p_3 = \dfrac{^4C_2 \times {^6C_1}}{^{10}C_3} = \dfrac{3}{10}$

When $X = 3$, $p_4 = \dfrac{^4C_3}{^{10}C_3} = \dfrac{1}{30}$

| $X$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $p_i$ | $\frac{1}{6}$ | $\frac{1}{2}$ | $\frac{3}{10}$ | $\frac{1}{30}$ |

Now, Mean($\mu$) = $\sum p_i X_i$

$= 0\times\dfrac{1}{6}+1\times\dfrac{1}{2}+2\times\dfrac{3}{10}+3\times\dfrac{1}{30} = \dfrac{1}{2}+\dfrac{6}{10}+\dfrac{1}{10} = \dfrac{6}{5}$

Variance = $\sum X_i^2 p_i - (\Sigma X_i p_i)^2$

$= \left(0\times\dfrac{1}{6}+1\times\dfrac{1}{2}+4\times\dfrac{3}{10}+9\times\dfrac{1}{30}\right)-\dfrac{36}{25}$

$= \left(\dfrac{1}{2}+\dfrac{12}{10}+\dfrac{9}{30}\right)-\dfrac{36}{25}$

$= 2-\dfrac{36}{25}=\dfrac{14}{25}$

**21.** Let $x$ grams of wheat and $y$ grams of rice should be mixed.

Therefore, we have

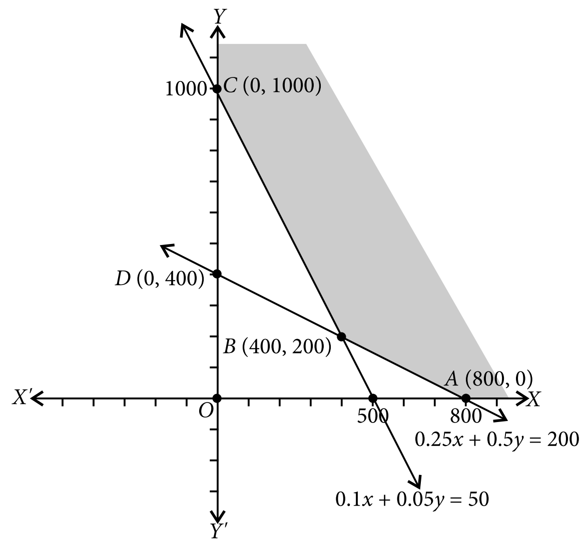
$0.1\,x + 0.05\,y \geq 50$ ...(i)

$0.25\,x + 0.5\,y \geq 200$ ...(ii)

$x, y \geq 0$ ...(iii)

and $Z_{\min} = \dfrac{4x}{1000}+\dfrac{6y}{1000}$

On plotting equations (i), (ii) and (iii), we have the following graph.




Here, the shaded region is the required region on the basis of constraints and which is unbounded (shown in figure). Now, we find the value of $Z$ at each corner point.

| Corner Point | Value of $Z$ | |
|---|---|---|
| $A$ (800, 0) | 3.2 | |
| $B$ (400, 200) | 2.8 | ← Minimum |
| $C$ (0, 1000) | 6 | |

Hence, cost will be minimum when 400 grams of wheat and 200 grams of rice are mixed in the daily diet, and minimum cost is ₹ 2.8 per gram.

Proteins are used as a building material. It stores and transport molecules.

**22.** Equation of the plane passing through (3, 4, 1) is

$a(x - 3) + b(y - 4) + c(z - 1) = 0$ ...(i)

Since this plane passes through (0, 1, 0) also

$\therefore$ $a(0 - 3) + b(1 - 4) + c(0 - 1) = 0$

or $-3a - 3b - c = 0$

or $3a + 3b + c = 0$ ...(ii)

Since (i) is parallel to the line

$$\frac{x+3}{2}=\frac{y-3}{7}=\frac{z-2}{5}$$

$\therefore$ $2a + 7b + 5c = 0$ ...(iii)

From (ii) and (iii), we get

$$\frac{a}{15-7}=\frac{b}{2-15}=\frac{c}{21-6}=k$$

$\Rightarrow$ $a = 8k, b = -13k, c = 15k$

Putting in (i), we have

$8k(x - 3) - 13k(y - 4) + 15k(z - 1) = 0$

$\Rightarrow \quad 8(x-3) - 13(y-4) + 15(z-1) = 0$

$\Rightarrow \quad 8x - 13y + 15z + 13 = 0$

which is the required equation of the plane.

**OR**

Let $\dfrac{x+2}{3} = \dfrac{2y+3}{4} = \dfrac{3z+4}{5} = \lambda$

Any general point on the line is given by

$3\lambda - 2, \dfrac{4\lambda-3}{2}, \dfrac{5\lambda-4}{3}$

Now, direction ratios if a point on the line is joined to (–2, 3, –4) are

$\Rightarrow \quad 3\lambda, \dfrac{4\lambda-9}{2}, \dfrac{5\lambda+8}{3}$

Now the distance is measured parallel to the plane

$4x + 12y - 3z + 1 = 0$

$\therefore \quad 4 \times 3\lambda + 12 \times \left(\dfrac{4\lambda-9}{2}\right) - 3 \times \left(\dfrac{5\lambda+8}{3}\right) = 0$

$\Rightarrow \quad 12\lambda + 24\lambda - 54 - 5\lambda - 8 = 0$

$\Rightarrow \quad 31\lambda - 62 = 0 \Rightarrow \quad \lambda = 2$

$\therefore$ The point required is $\left(4, \dfrac{5}{2}, 2\right)$.

$\therefore$ Distance $= \sqrt{(4+2)^2 + \left(\dfrac{5}{2}-3\right)^2 + (2+4)^2}$

$= \sqrt{36+36+\dfrac{1}{4}} = \sqrt{\dfrac{289}{4}} = \dfrac{17}{2}$ units

**23.** Let the length and breadth of the tank be $L$ and $B$.

$\therefore$ Volume $= 8 = 2\,LB \Rightarrow B = \dfrac{4}{L}$ …(i)

The surface area of the tank, $S$ = Area of Base + Area of 4 Walls

$= LB + 2(B+L)^2 = LB + 4B + 4L$

The cost of constructing the tank is

$C = 70(LB) + 45(4B + 4L)$

$= 70\left(L.\dfrac{4}{L}\right) + 180\left(\dfrac{4}{L} + L\right)$

$\Rightarrow \quad C = 280 + 180\left(\dfrac{4}{L} + L\right)$ …(ii)

Differentiating (ii) both sides w.r.t. $L$, we get

$\dfrac{dC}{dL} = -\dfrac{720}{L^2} + 180$ …(iii)

For minimisation, $\dfrac{dC}{dL} = 0$

$\Rightarrow \quad \dfrac{720}{L^2} = 180 \Rightarrow L^2 = \dfrac{720}{180} = 4 \Rightarrow L = 2$

Differentiating (iii) again w.r.t. $L$, we get

$\dfrac{d^2C}{dL^2} = \dfrac{1440}{L^3} > 0 \ \forall\ L > 0$

$\therefore$ Cost is minimum when $L = 2$

From (i), $B = 2$

Minimum cost $= 280 + 180\left(\dfrac{4}{2} + 2\right)$ (from (ii))

$= 280 + 720 = ₹\ 1000$

**OR**

Let $O$ be the centre of sphere of radius 12 cm and a cone $ABC$ of radius $R$ cm and $h$ cm inscribed in the sphere.

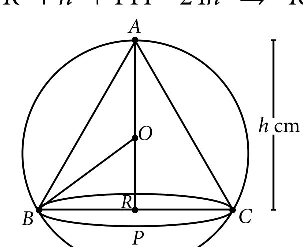
$AP = AO + OP \Rightarrow h = 12 + OP \Rightarrow OP = h - 12$

Now in right angled $\Delta OBP$, we have

$BO^2 = BP^2 + OP^2$

$\Rightarrow \quad (12)^2 = R^2 + (h-12)^2$

$\Rightarrow \quad 144 = R^2 + h^2 + 144 - 24h \Rightarrow R^2 = 24h - h^2$




Volume of cone, $V = \dfrac{1}{3}\pi R^2 h = \dfrac{1}{3}\pi\left(24h - h^2\right)h$

$\Rightarrow \quad V = \dfrac{1}{3}\pi\left(24h^2 - h^3\right) \Rightarrow \dfrac{dV}{dh} = \dfrac{\pi}{3}\left(48h - 3h^2\right)$

For maximum/minimum value of $V$, we have

$\dfrac{dV}{dh} = 0 \quad \Rightarrow 48h - 3h^2 = 0$

$\Rightarrow \quad h(48 - 3h) = 0 \Rightarrow$ Either $h = 0$ or $h = 16$

But height of cone cannot be zero.

Therefore $h = 16$ cm.

Now, $\dfrac{d}{dh}\left(\dfrac{dV}{dh}\right) = \dfrac{\pi}{3}(48 - 6h)$

$\Rightarrow \quad \left(\dfrac{d^2V}{dh^2}\right)_{h=16} = \dfrac{\pi}{3}(48 - 6 \times 16) = -16\pi < 0$

Hence, volume of cone is maximum when $h = 16$ cm

**24.** We have $x\dfrac{dy}{dx} + y - x + xy\cot x = 0, x \neq 0$

we have $\dfrac{dy}{dx} + \left(\cot x + \dfrac{1}{x}\right)y = 1$ …(i)

This is a linear equation of the form $\frac{dy}{dx} + Py = Q$

Here, $P = \cot x + \frac{1}{x}$, $Q = 1$

So, I.F. $= e^{\int\left(\cot x+\frac{1}{x}\right)dx} = e^{\log|\sin x| + \log|x|}$

$= e^{\log|x\sin x|} = |x \sin x| = x \sin x$

($x \sin x$ is always +ve for any $x$)

Solution is given by

$$y \times \text{I.F.} = \int Q \times \text{I.F.}\ dx$$

$\Rightarrow\ y \times x \sin x = \int 1 \times x \sin x\ dx$

$\Rightarrow\ xy \sin x = \int x \sin x\ dx$

$\Rightarrow\ xy \sin x = x(-\cos x) - \int -\cos x \times 1\ dx$

$\Rightarrow\ xy \sin x = -x \cos x + \sin x + c$

$\therefore\ xy \sin x = \sin x - x \cos x + c$

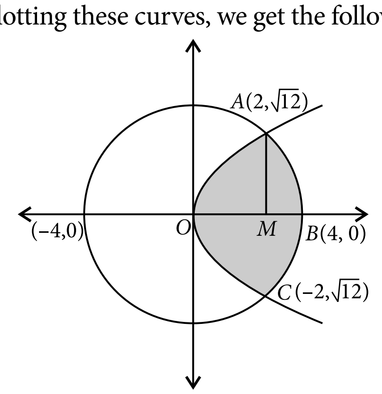
Hence, $y = \frac{1}{x} - \cot x + \frac{C}{x \sin x}$ is the required solution.

**25.** We have, $y^2 = 6x$ ...(i)

and $x^2 + y^2 = 16$ ...(ii)

Area of the circle $= \pi \times 16 = 16\pi$ sq. units

On plotting these curves, we get the following figure.




Now, Area of the shaded region

$= 2$(Area of $OAM$ + Area of $AMB$)

$$= 2\left[\int_0^2 \sqrt{6x}\,dx + \int_2^4 \sqrt{16-x^2}\,dx\right]$$

$$= 2\left[\sqrt{6}\left[\frac{x^{3/2}}{3/2}\right]_0^2 + \left[\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_2^4\right]$$

$$= 2\left[\frac{2}{3}\sqrt{6}\left(2^{3/2}\right) + 8.\frac{\pi}{2} - \sqrt{12} - 8 \times \frac{\pi}{6}\right]$$

$$= 2\left[\frac{2}{3}\sqrt{48} - \sqrt{12} + 4\pi - \frac{4\pi}{3}\right]$$

$$= 2\left[\frac{2}{3} \times 4\sqrt{3} - 2\sqrt{3} + \frac{8\pi}{3}\right]$$

$$= 2\left(\frac{2\sqrt{3}}{3} + \frac{8\pi}{3}\right)$$

$$= \frac{4\sqrt{3}}{3} + \frac{16\pi}{3}$$

$\therefore$ Area of unshaded region $= 16\pi - \left(\frac{4\sqrt{3}}{3} + \frac{16\pi}{3}\right)$

$\therefore$ Required ratio

$$= \left(\frac{4\sqrt{3}}{3} + \frac{16\pi}{3}\right) \Big/ 16\pi - \left(\frac{4\sqrt{3}}{3} + \frac{16\pi}{3}\right)$$

$$= \frac{\frac{4\sqrt{3}}{3} + \frac{16\pi}{3}}{\frac{32\pi}{3} - \frac{4\sqrt{3}}{3}} = \frac{\frac{1}{\sqrt{3}} + \frac{4\pi}{3}}{\frac{8\pi}{3} - \frac{1}{\sqrt{3}}}$$

$$= \frac{3 + 4\sqrt{3}\pi}{8\sqrt{3}\pi - 3}$$

Hence required ratio is $(3 + 4\sqrt{3}\pi) : (8\sqrt{3}\pi - 3)$

**26.** Given $f(x) = 9x^2 + 6x - 5$

Let $x_1, x_2 \in R_+$, such that $f(x_1) = f(x_2)$

$\Rightarrow\ 9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$

$\Rightarrow\ 9(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0$

$\Rightarrow\ (x_1 - x_2)\{9(x_1 + x_2) + 6\} = 0$

$\Rightarrow\ x_1 - x_2 = 0$ as $9(x_1 + x_2) + 6 > 0$

$\therefore\ x_1 = x_2$

So, $f(x)$ is one-one and we see that for every $x \in R_+$, there is $f(x) \in [-5, \infty)$

So, $f(x)$ is onto.

*i.e.*, $f(x)$ is one-one and onto. Hence $f(x)$ is invertible.

Now, let $y = 9x^2 + 6x - 5$

$\Rightarrow\ 9x^2 + 6x - 5 - y = 0$

$$\Rightarrow\ x = \frac{-6 \pm \sqrt{36 + 4 \times 9(5+y)}}{2 \times 9}$$

As $x \in R_+$

$$\therefore\ x = \frac{-6 + 6\sqrt{1 + 5 + y}}{18} = \frac{-6 + 6\sqrt{6+y}}{18}$$

$$= \frac{-1 + \sqrt{6+y}}{3}$$

$$\therefore\ f^{-1}(y) = \frac{-1 + \sqrt{6+y}}{3}$$

# WB JEE MOCK TEST PAPER

## CATEGORY-I

**For each correct answer one mark will be awarded, whereas, for each wrong answer, 25% of total marks (1/4) will be deducted. If candidates mark more than one answer, negative marking will be done.**

1. Ten different letters of an alphabet are given. Words with five letters are formed from these given letters. Then the number of words which have at least one letter repeated is
(a) 69,760 (b) 24,320
(c) 99,777 (d) none of these

2. The interior angles of a regular polygon measure 120° each. The number of diagonals of the polygon is
(a) 9 (b) 15 (c) 44 (d) 33

3. If $(\cot^{-1}x)^2 - 7(\cot^{-1}x) + 10 > 0$, then $x$ lies in the interval
(a) (cot5, cot2) (b) $(-\infty, \cot 5) \cup (\cot 2, \infty)$
(c) $(-\infty, \cot 5)$ (d) $(\cot 2, \infty)$

4. The degree of the expansion
$$(x+(x^3-1)^{1/2})^5+(x-(x^3-1)^{1/2})^5 \text{ is}$$
(a) 15 (b) 7 (c) 6 (d) 5

5. If $z_1, z_2, z_3$ represent the vertices of a triangle, then the centroid of the triangle is given by
(a) $\dfrac{az_1+bz_2+cz_3}{a+b+c}$ (b) $\dfrac{z_1+z_2+z_3}{3}$
(c) $\dfrac{z_1z_2z_3}{3}$ (d) none of these

6. If $0 < \alpha < \pi$ then the quadratic equation $\cos(\alpha - 1)x^2 + x\cos\alpha + \sin\alpha = 0$, has
(a) both roots imaginary
(b) only one root imaginary
(c) only one root irrational
(d) none of these

7. The expression $\dfrac{1}{\tan x+\cot x+\sec x+\operatorname{cosec} x}$ equivalent to
(a) $\dfrac{1}{2(\sin x+\cos x-1)}$ (b) $\dfrac{(\sin x+\cos x-1)}{2}$
(c) $\dfrac{1}{2(\sin x-\cos x+1)}$ (d) $\dfrac{(\sin x-\cos x+1)}{2}$

8. If $\theta \in \left(\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$ and $f(\theta) = \sec 2\theta - \tan 2\theta$, then
$$f\left(\frac{\pi}{4}-\theta\right)=$$
(a) $\tan\theta$ (b) $\cot\theta$
(c) $\sec 2\theta$ (d) $\tan 2\theta$

9. The maximum value of $\dfrac{a\cos A+b\cos B+c\cos C}{a+b+c}$, where $a$, $b$, $c$ and $A$, $B$, $C$ have their usual meaning, is
(a) $\sin A$ (b) $4\sin\dfrac{A}{2}\sin\dfrac{B}{2}\sin\dfrac{C}{2}$
(c) $\sin\dfrac{C}{2}$ (d) none of these

10. $\displaystyle\lim_{x\to 0}\frac{f(x)}{x}=2$ where $f(x)=\min\left\{\sin\sqrt{[m]}x, |x|\right\}$ and $[\cdot]$ greatest integer, then
(a) $m \in \{4\}$ (b) $m \in [4, 5]$
(c) $m \in [4, 5)$ (d) $m \in \{5\}$

11. If $\displaystyle\int_0^x f(z)dz = x+\int_x^1 zf(z)\,dz$ then $\displaystyle\int_1^2 f(x)dx$ equals

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(a) $1 + x$ (b) $\log\left(\frac{2}{3}\right)$

(c) $\log 3$ (d) $\log\left(\frac{3}{2}\right)$

**12.** A line makes the same angle $\theta$ with each of the $x$ and $z$ axis. If the angle $\beta$, which it makes with $y$-axis is such that $\sin^2\beta = 3\sin^2\theta$ then $\cos^2\theta$ equals

(a) 3/5 (b) 1/5 (c) 2/3 (d) 2/5

**13.** $\int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx = \frac{k}{4}$, then the value of $k$ equals

(a) $\pi/12$ (b) $\pi/3$

(c) $\pi/2$ (d) none of these

**14.** $\lim_{x\to 0} \frac{\log(1+x^3)}{\sin^3 x}$ equals

(a) 1 (b) 0

(c) $-1$ (d) none of these

**15.** $\cos^2 A + \cos^2(B - A) - 2\cos A \cos B \cos(A - B) =$

(a) $\cos 2A$ (b) $\sin^2 A$

(c) $\sin^2 B$ (d) $\cos^2 B$

**16.** The domain of the function

$f(x) = \log_4(\log_5(\log_3(18x - x^2 - 77)))$ is

(a) $x \in (4, 5)$ (b) $x \in (0, 10)$

(c) $x \in (8, 10)$ (d) none of these

**17.** The order and degree of the differential equation of all tangent lines to the parabola $x^2 = 4y$ is

(a) 2, 2 (b) 3, 1 (c) 1, 2 (d) 4, 1

**18.** The area enclosed between the curves $y^2 = x$ and $y = |x|$ is

(a) 1/6 (b) 1/3 (c) 2/3 (d) 1

**19.** Let $f$ be the differentiable for $\forall\ x$. If $f(1) = -2$ and $f'(x) \geq 2$ for [1, 6], then

(a) $f(6) < 8$ (b) $f(6) \geq 8$

(c) $f(6) = 5$ (d) $f(6) < 5$

**20.** If $xy = e - e^y$ then $\frac{d^2y}{dx^2}$ equals when $x = 0$

(a) $1/e$ (b) $1/e^3$

(c) $1/e^2$ (d) none of these

**21.** $\int \cos\sqrt{x}\,dx$ is equal to

(a) $-\frac{\sin\sqrt{x}}{2\sqrt{x}} + c$

(b) $\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x} + c$

(c) $2(\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}) + c$

(d) $2(\sqrt{x}\sin\sqrt{x} - \cos\sqrt{x}) + c$

**22.** If $f(x + y) = f(x) + f(y) - xy - 1\ \forall\ x, y \in R$ and $f(1) = 1$, then the number of solutions of $f(n) = n, n \in N$ is

(a) 0 (b) 1

(c) 2 (d) none of these

**23.** In a G.P. of positive terms , any term is equal to the sum of next two terms, then common ratio of this G.P. is

(a) cos18° (b) sin18°

(c) 2cos18° (d) 2sin18°

**24.** Solution set of inequality $\log_{10}(x^2 - 2x - 2) \leq 0$ is

(a) $[-1,\ 1-\sqrt{3}]$

(b) $[1+\sqrt{3},\ \sqrt{3}]$

(c) $[-1,\ 1-\sqrt{3}) \cup (1+\sqrt{3}, 3]$

(d) none of these

**25.** In a survey it is to be found that 70% of employees like bananas and 64% like apples. If $x$% like both bananas and apples, then

(a) $x \geq 34$ (b) $x \leq 64$

(c) $34 \leq x \leq 64$ (d) all of these

**26.** If a relation $R$ is defined from a set $A = \{2, 3, 4, 5\}$ to a set $B = \{3, 6, 7, 10\}$ as follows $(x, y) \in R \Leftrightarrow x$ divides $y$. Expression of $R^{-1}$ is represented by

(a) {(6, 2), (10, 2), (3, 3)}

(b) {(6, 2), (10, 5), (3, 3)}

(c) {(6, 2), (10, 2), (3, 3), (6, 3), (10, 5)}

(d) none of these

**27.** If two arithmetic means $A_1, A_2$, two geometric means $G_1, G_2$ and two harmonic means $H_1, H_2$ are inserted between any two numbers then $\frac{A_1 + A_2}{H_1 + H_2}$ is

(a) $\frac{G_1G_2}{H_1H_2}$ (b) $\sqrt{G_1G_2}$

(c) $\frac{H_1H_2}{G_1G_2}$ (d) none of these

**28.** A test consist of 6 multiple choice questions each having four alternative answers of which only one is correct. Also only one of the alternatives must be marked by each candidate. The number of ways of getting exactly four correct answers by a candidate answering all the questions is

(a) $4^6 - 3^2$ (b) 135 (c) 55 (d) 120

29. An ellipse of eccentricity $\frac{2\sqrt{2}}{3}$ is inscribed in a circle and a point within the circle is chosen at random. The probability that this point lies outside the ellipse is

(a) 1/9 (b) 4/9 (c) 1/3 (d) 2/3

30. Consider points $A(3, 4)$ and $B(7, 13)$. If $P$ be a point on line $y = x$ such that $PA + PB$ is minimum, then co-ordinate of $P$ is

(a) $\left(\frac{2}{7}, \frac{12}{7}\right)$ (b) $\left(\frac{13}{7}, \frac{13}{7}\right)$

(c) $\left(\frac{23}{7}, \frac{23}{7}\right)$ (d) none of these

31. The equation $(x + y - 6)(xy - 3x - y + 3) = 0$ represents the sides of a triangle then the equation of the circumcircle of the triangle is

(a) $x^2 + y^2 - 5x - 9y + 20 = 0$

(b) $x^2 + y^2 - 4x - 8y + 18 = 0$

(c) $x^2 + y^2 - 3x - 5y + 8 = 0$

(d) $x^2 + y^2 + 2x - 3y - 1 = 0$

32. If the difference between the roots of the equation $ax^2 + ax + b = 0$ is equal to the difference between the roots of the equation $x^2 + bx + a = 0$ $(a \neq b)$, then

(a) $a + b = 4$ (b) $a + b = -4$

(c) $a - b = 4$ (d) $a - b = -4$

33. If $\Delta = \begin{vmatrix} \cos\frac{\theta}{2} & 1 & 1 \\ 1 & \cos\frac{\theta}{2} & -\cos\frac{\theta}{2} \\ -\cos\frac{\theta}{2} & 1 & 1 \end{vmatrix}$ lies in the interval

(a) [2, 4] (b) [0, 4] (c) [1, 3] (d) [–2, 2]

34. The values of $\alpha$ for which the system of equations $\alpha x - 3y + z = 0, x + \alpha y + 3z = 1, 3x + y + 5z = 2$, does not have unique solution are

(a) $-1, \frac{11}{5}$ (b) $-1, \frac{-11}{5}$

(c) $1, \frac{-11}{5}$ (c) $1, \frac{11}{5}$

35. The equation of the plane perpendicular to the line $\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z+1}{2}$ and passing through the point (2, 3, 1) is

(a) $\vec{r}\cdot(\hat{i}+\hat{j}+2\hat{k})=1$ (b) $\vec{r}\cdot(\hat{i}-\hat{j}+2\hat{k})=1$

(c) $\vec{r}\cdot(\hat{i}-\hat{j}+2\hat{k})=7$ (d) none of these

36. $\vec{a}, \vec{b}, \vec{c}$ are three vectors of equal magnitude. The angle between each pair of vectors is $\pi/3$ such that $|\vec{a}+\vec{b}+\vec{c}|=\sqrt{6}$ then $|\vec{a}|$ is equal to

(a) 2 (b) –1 (c) 1 (d) $\frac{1}{3}\sqrt{6}$

37. The coordinates of a point on the parabola $y^2 = 8x$ whose focal distance is 4 are

(a) $\left(\frac{1}{2}, \pm 2\right)$ (b) $(1, \pm 2\sqrt{2})$

(c) $(2, \pm 4)$ (d) $(\pm 2, 4)$

38. The derivative of the function

$$f(x)=\cos^{-1}\left\{\frac{1}{\sqrt{13}}(2\cos x-3\sin x)\right\}$$
$$+\sin^{-1}\left(\frac{1}{\sqrt{13}}(2\sin x+3\cos x)\right)$$

with respect to $\sqrt{1+x^2}$ is

(a) $2x$ (b) $2\sqrt{1+x^2}$

(c) $\frac{2}{x}\sqrt{1+x^2}$ (d) $\frac{2x}{\sqrt{1+x^2}}$

39. Let $f''(x)$ be continuous at $x = 0$ and $f''(0) = 4$. The value of $\lim\limits_{x\to 0}\frac{2f(x)-3f(2x)+f(4x)}{x^2}$ is equal to

(a) 11 (b) 2

(c) 12 (d) none of these

40. If $b_1, b_2, b_3, \ldots$ belongs to A.P. such that $b_1 + b_4 + b_7 + \ldots + b_{28} = 220$ then the value of $b_1 + b_2 + b_3 + \ldots + b_{28} =$

(a) 616 (b) 308

(c) 2,200 (d) 1,232

41. The middle term in the expansion of $(1 - 3x + 3x^2 - x^3)^{2n}$ is

(a) $\frac{6n!}{3n!3n!}x^n$ (b) $\frac{6n!}{3n!}x^{3n}$

(c) $\frac{6n!}{3n!3n!}(-x)^{3n}$ (d) none of these

42. The solution of the differential equation $(x^2 + y^2)dx - 2xydy = 0$ is

(a) $\frac{x}{x^2+y^2}=c$ (b) $\frac{x^2+y^2}{x}=c$

(c) $\frac{y^2-x^2}{x}=c$ (d) $\frac{x^2-y^2}{x}=c$

**43.** The number of vectors of unit length perpendicular to vector $\vec{a} \equiv (5, 6, 0)$ and $\vec{b} \equiv (6, 5, 0)$ is

(a) 1 (b) 4 (c) 3 (d) 2

**44.** If the eccentricity of the hyperbola $x^2 - y^2\sec^2\theta = 4$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2\sec^2\theta + y^2 = 16$ the value of $\theta$ equals

(a) $\pi/6$ (b) $3\pi/4$ (c) $\pi/3$ (d) $\pi/2$

**45.** Angle subtended by common tangents of two ellipses $4(x-4)^2 + 25y^2 = 100$ and $4(x+1)^2 + y^2 = 4$ at origin is

(a) $\pi/3$ (b) $\pi/4$
(c) $\pi/2$ (d) none of these

**46.** If $\frac{5z_2}{11z_1}$ is purely imaginary, then $\left|\frac{2z_1+3z_2}{2z_1-3z_2}\right|$ is

(a) $\frac{37}{33}$ (b) $\frac{11}{5}$ (c) 1 (d) $\frac{5}{11}$

**47.** If the trace of the matrix

$$A = \begin{pmatrix} x-5 & 0 & 2 & 4 \\ 3 & x^2-10 & 6 & 1 \\ -2 & 3 & x-7 & 1 \\ 1 & 2 & 0 & -2 \end{pmatrix}$$

assumes the value zero, then the value of $x$ equals to,

(a) $-6, -4$ (b) $-6, 4$
(c) $6, 4$ (d) $6, -4$

**48.** Let $\phi(x) = f(x) + f(1-x)$ and $f''(x) < 0$ in $0 \le x < 1$, then

(a) $\phi(x)$ decreases in $(0, 1)$
(b) $\phi(x)$ increases in $(0, 1)$
(c) $\phi(x)$ decreases in $(0, 1/2)$
(d) none of these

**49.** The number of tangents to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ which are equally inclined to the axes is

(a) 4 (b) 3 (c) 2 (d) 1

**50.** The odds against $A$ solving a certain problem are 3 to 2 and the odds in favour of $B$ solving the same are 2 to 1. The probability that the problem will be solved if they both try, is

(a) $\frac{2}{5}$ (b) $\frac{11}{15}$ (c) $\frac{4}{5}$ (d) $\frac{2}{3}$

**51.** Six coins are tossed simultaneously. The probability atleast one tail turns up is

(a) $\frac{63}{64}$ (b) $\frac{1}{64}$
(c) $\frac{3}{32}$ (d) none of these

**52.** It is given that event $A$ and $B$ are such that $P(A) = \frac{1}{4}, P(A|B) = \frac{1}{2}, P(B|A) = \frac{2}{3}$ then $P(B)$ is equal to

(a) 1/3 (b) 2/3 (c) 1/2 (d) 1/6

**53.** Let $P(n): 2^n > n \,\forall\, n \in N$ and $2^k > k, \forall\, n = k$, then which of the following is true? $(k \ge 2)$

(a) $2^k > 5k > 1$ (b) $2^{k+1} > 2k > k+1$
(c) $2^k > 2(k+1) > k$ (d) none of these

**54.** If $z \neq 0$, then $\int_0^{50} \arg(-|z|)\,dx =$

(a) 50 (b) not defined
(c) 0 (d) $50\pi$

**55.** The number of values of $\theta$ in $[0, 2\pi]$ that satisfy the equation $3\cos 2\theta + 13\sin\theta - 8 = 0$ is

(a) 1 (b) 2 (c) 3 (d) 4

**56.** If $A = \{a, b, c, d\}$ and $B = \{x, y, z\}$, then which one of the following relations from $A$ to $B$ is not a mapping?

(a) $\{(a, x), (b, y), (c, z), (d, x)\}$
(b) $\{(a, y), (b, y), (c, x), (d, z)\}$
(c) $\{(b, x), (c, x), (d, z), (a, y)\}$
(d) $\{(b, x), (a, y), (b, z), (c, z)\}$

**57.** In a moderately asymmetrical distribution, the mean is 18 and median 22, the value of mode is

(a) 30 (b) 10
(c) 4 (d) none of these

**58.** If $\vec{P} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{Q} = \hat{i} + 3\hat{k}$. If $\vec{R}$ is a unit vector then the maximum value of the scalar triple product $[\vec{R}\ \vec{P}\ \vec{Q}]$ is

(a) $-1$ (b) $\sqrt{10} + \sqrt{6}$
(c) $\sqrt{59}$ (d) $\sqrt{60}$

**59.** If the coefficient of variation of some observations is 60 and their standard deviation is 20, then their mean is

(a) 35 (b) 34
(c) 38.3 (d) 33.33

**60.** If ω is a cube root of unity then

$$\tan\left\{\left(\omega^{200}+\frac{1}{\omega^{200}}\right)\pi+\frac{\pi}{4}\right\}=$$

(a) 1 (b) $\frac{1}{\sqrt{2}}$

(c) 0 (d) none of these

## CATEGORY-II

**Every correct answer will yield 2 marks. For incorrect response, 25% of full mark (1/2) would be deducted. If candidates mark more than one answer, negative marking will be done.**

**61.** If $m$ is A.M. of two distinct real numbers $l$ and $n$ $(l, n > 1)$ and $G_1$, $G_2$, and $G_3$ are three geometric means between $l$ and $n$, then $G_1^4+2G_2^4+G_3^4$ equals

(a) $4lmn^2$ (b) $4l^2m^2n^2$

(c) $4l^2mn$ (d) $4lm^2n$

**62.** Let α and β be the roots of the equation $x^2-6x-2=0$. If $a_n=\alpha^n-\beta^n$, for $n \ge 1$, then the value of $\frac{a_{10}-2a_8}{2a_9}=$

(a) 3 (b) –3 (c) 6 (d) –6

**63.** The relation $R$ in $N \times N$ such that $(a, b)\ R\ (c, d)$ iff $a+d=b+c$, is

(a) reflexive but not symmetric

(b) reflexive and transitive but not symmetric

(c) an equivalence relation

(d) none of these

**64.** In a test, an examinee either guesses or copies or knows the answer to a multiple choice question with four choices and only one correct option. The probability that he makes a guess is 1/3. The probability that he copies the answer is 1/6. The probability that the answer is correct, given that he copied it, is 1/8. The probability that he knows the answer to the question , given that he correctly answered it is

(a) $\frac{23}{29}$ (b) $\frac{27}{29}$ (c) $\frac{24}{29}$ (d) $\frac{25}{29}$

**65.** Let $\tan^{-1}y=\tan^{-1}x+\tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $|x|<\frac{1}{\sqrt{3}}$. The value of $y$ is

(a) $\frac{3x-x^3}{1+3x^2}$ (b) $\frac{3x+x^3}{1+3x^2}$

(c) $\frac{3x-x^3}{1-3x^2}$ (d) $\frac{3x+x^3}{1-3x^2}$

**66.** Let $a$ and $b$ be two non-zero reals such that $a \ne b$. Then the equation of the line passing through origin and point of intersection of $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{b}+\frac{y}{a}=1$ is

(a) $ax+by=0$ (b) $bx+ay=0$

(c) $y-x=0$ (d) $x+y=0$

**67.** If $f(x)=\frac{\sin(2\pi[\pi^2x])}{5+[x]^2}$, ([·] denotes the greatest integer function), Then $f(x)$ is

(a) discontinuous at some $x$

(b) continuous at all $x$, but the derivative $f'(x)$ doesn't exist for some $x$

(c) $f''(x)$ does not exist for all $x$

(d) none of these

**68.** $\int\frac{dx}{x^{22}(x^7-6)}=A\{\ln(p)^6+9p^2-2p^3-18p\}+c$ then

(a) $A=\frac{1}{9072}, p=\left(\frac{x^7-6}{x^7}\right)$

(b) $A=\frac{1}{54432}, p=\left(\frac{x^7-6}{x^7}\right)$

(c) $A=\frac{1}{54432}, p=\left(\frac{x^7}{x^7-6}\right)$

(d) $A=\frac{1}{9072}, p=\left(\frac{x^7-6}{x^7}\right)^{-1}$

**69.** Solution of $\left(\frac{x+y-1}{x+y-2}\right)\frac{dy}{dx}=\left(\frac{x+y+1}{x+y+2}\right)$, given that $y=1$ when $x=1$, is

(a) $\log\left|\frac{(x-y)^2-2}{2}\right|=2(x+y)$

(b) $\log\left|\frac{(x-y)^2+2}{2}\right|=2(x-y)$

(c) $\log\left|\frac{(x+y)^2+2}{2}\right|=2(x-y)$

(d) none of these

**70.** The minimum value of $px+qy$ when $xy=r^2$ is equal to

(a) $2r\sqrt{pq}$ (b) $2pr\sqrt{r}$

(c) $-2r\sqrt{pq}$ (d) none of these

CATEGORY-III

**In this section more than 1 answer can be correct. Candidates will have to mark all the correct answers, for which 2 marks will be awarded. If, candidates marks one correct and one incorrect answer then no marks will be awarded. But if, candidate makes only correct, without making any incorrect, formula below will be used to allot marks. 2 × (no. of correct response/total no. of correct options)**

**71.** If the vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are any four vectors then $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})$ is equal to

(a) $\vec{a}\cdot\{b\times(\vec{c}\times\vec{d})\}$

(b) $(\vec{a}\cdot\vec{c})(\vec{b}\cdot\vec{d})-(\vec{a}\cdot\vec{d})(\vec{b}\cdot\vec{c})$

(c) $\{(\vec{a}\times\vec{b})\times\vec{c}\}\cdot\vec{d}$

(d) $(\vec{d}\times\vec{c})\cdot(\vec{b}\times\vec{a})$

**72.** Let $z_1$, $z_2$ be two complex numbers represented by points on the circle $|z_1| = 1$ and $|z_2| = 2$ respectively then

(a) $\max|2z_1 + z_2| = 4$

(b) $\min|z_1 - z_2| = 1$

(c) $\left|z_2+\frac{1}{z_1}\right|\le 3$

(d) none of these

**73.** In $\Delta ABC$, if $\cos\frac{A}{2}=\sqrt{\frac{b+c}{2c}}$, then

(a) area of the triangle is $\frac{1}{2}ab$

(b) circumradius is equal to $\frac{1}{2}c$

(c) area of the triangle is $\frac{1}{2}bc$

(d) circumradius is equal to $\frac{1}{2}a$

**74.** In the expansion of $(x + y + z)^{25}$

(a) every term is of the form ${}^{25}C_r\cdot{}^{r}C_k\cdot x^{25-r}\cdot y^{r-k}\,z^k$

(b) the coefficient of $x^8y^9z^9$ is zero

(c) the number of terms is 325

(d) none of these

**75.** $\begin{vmatrix} x^2 & (y+z)^2 & yz \\ y^2 & (z+x)^2 & zx \\ z^2 & (x+y)^2 & xy \end{vmatrix}$ is divisible by

(a) $x^2 + y^2 + z^2$ (b) $x - y$

(c) $x - y - z$ (d) $x + y + z$

**76.** A function $f(x)$ is defined in the interval [1, 4] as follows:

$$f(x)=\begin{cases}\log_e[x], & 1\le x<3\\ |\log_e x|, & 3\le x<4\end{cases}$$

The graph of the function $f(x)$

(a) is broken at two points

(b) is broken at exactly at one point

(c) does not have a definite tangent at two points

(d) does not have a definite tangent at more than two points.

**77.** A coin is tossed repeatedly. $A$ and $B$ call alternately for winning a prize of ₹ 30. One who calls correctly first wins the prize. $A$ starts the call. Then the expectation of

(a) $A$ is ₹ 10 (b) $B$ is ₹ 10

(c) $A$ is ₹ 20 (d) $B$ is ₹ 20

**78.** The function $f(x)=x^2+\frac{\lambda}{x}$ has a

(a) minimum at $x = 2$ if $\lambda = 16$

(b) maximum at $x = 2$ if $\lambda = 16$

(c) maximum for no real value of $\lambda$

(d) point of inflection at $x = 1$ if $\lambda = -1$

**79.** Let $I_n=\int_0^{\pi/4}\tan^n x\,dx, n\in N$, Then

(a) $I_1 = I_3 + 2I_5$ (b) $I_n+I_{n-2}=\frac{1}{n}$

(c) $I_n+I_{n-2}=\frac{1}{n-1}$ (d) none of these

**80.** Let $f(x) = x^2 + xg'(1) + g''(2)$ and $g(x) = f(1)\cdot x^2 + xf'(x) + f''(x)$ then

(a) $f'(1) + f'(2) = 0$ (b) $g'(2) = g'(1)$

(c) $g''(2) + f''(3) = 6$ (d) none of these

SOLUTIONS

**1. (a) :** Total number of 5 letter word when any letter can be chosen any number of times is $(10)^5$.

Now these numbers include those which never comes in the picture and such numbers are ${}^{10}P_5$.

∴ Required number of words which have at least one letter repeated are

$10^5 - {}^{10}P_5 = 1{,}00{,}000 - 30{,}240 = 69{,}760$

**2. (a) :** Let the number of sides of the polygon be $n$.

∴ $(2n - 4)90° = 120°n$

⇒ $180°n - 120°n = 360°$ ⇒ $60°n = 360°$

∴ $n = 6$

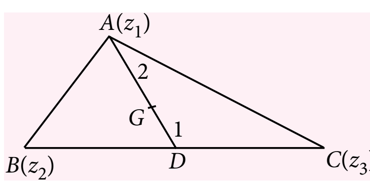
$\therefore$ number of diagonals = ${}^nC_2 - n$
$= {}^6C_2 - 6 = 15 - 6 = 9$

**3. (b) :** $(\cot^{-1}x)^2 - 7(\cot^{-1}x) + 10 > 0$
$\Rightarrow (\cot^{-1}x - 2)(\cot^{-1}x - 5) > 0$
$\Rightarrow \cot^{-1}x < 2$ and $\cot^{-1}x > 5$
But $\cot^{-1}x$ is decreasing $\forall\ x \in R$
$\Rightarrow x > \cot 2$ and $x < \cot 5$
$\Rightarrow x \in (-\infty, \cot 5) \cup (\cot 2, \infty)$

**4. (b) :** $[x + (x^3 - 1)^{1/2}]^5 + [x - (x^3 - 1)^{1/2}]^5$
$= 2[x^5 + 10x^3((x^3 - 1)^{1/2})^2) + 5x((x^3 - 1)^{1/2})^4$
[Note that : $(x + a)^n + (x - a)^n = 2[{}^nC_0\, x^n a^0 + {}^nC_2 x^{n-2} a^2 + {}^nC_4\, x^{n-4} a^4 + ...]$
$= 2[x^5 + 10x^3(x^3 - 1) + 5x(x^3 - 1)^2]$
$= 2[x^5 + 10x^6 - 10x^3 + 5x(x^6 - 2x^3 + 1)]$
$\therefore$ degree is 7.

**5. (b) :** $D$ is the mid-point of $BC$.
$\therefore\ BD = DC$

$$D = \frac{z_2 + z_3}{2}$$




We know that centroid divides median internally in 2 : 1 ratio.

$$\therefore\ G = \frac{2\cdot\frac{z_2 + z_3}{2} + z_1}{2 + 1} = \frac{z_1 + z_2 + z_3}{3}$$

**6. (d) :** Here the given equation is
$(\cos\alpha - 1)x^2 + x\cos\alpha + \sin\alpha = 0$
$\therefore\ D = B^2 - 4AC = \cos^2\alpha + 4\sin\alpha(1 - \cos\alpha)$

$> 0\ \forall\ 0 < \alpha < \pi$
$\therefore$ roots are real and distinct.

**7. (b) :** $$f(x) = \frac{1}{\tan x + \cot x + \sec x + \operatorname{cosec} x}$$

$$= \frac{\sin x \cos x}{1 + \sin x + \cos x} = \frac{\sin x}{(1 + \tan x) + \sec x}$$

$$= \frac{\sin x(1 + \tan x - \sec x)}{(1 + \tan x)^2 - \sec^2 x}$$

$$= \frac{\cos x(1 + \tan x - \sec x)}{2} = \frac{\sin x + \cos x - 1}{2}$$

**8. (a) :** $f(\theta) = \sec 2\theta - \tan 2\theta$

$$= \frac{1 - \sin 2\theta}{\cos 2\theta} = \frac{(\cos\theta - \sin\theta)^2}{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}$$

$$= \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1 - \tan\theta}{1 + \tan\theta} = \tan\left(\frac{\pi}{4} - \theta\right)$$

$$\therefore\ f\left(\frac{\pi}{4} - \theta\right) = \tan\left(\frac{\pi}{4} - \frac{\pi}{4} + \theta\right) = \tan\theta$$

**9. (b) :** $$\frac{a\cos A + b\cos B + c\cos C}{a + b + c}$$

$$\because\ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

$$= \frac{2R\sin A\cos A + 2R\sin B\cos B + 2R\sin C\cos C}{2R\sin A + 2R\sin B + 2R\sin C}$$

$$= \frac{R(\sin 2A + \sin 2B + \sin 2C)}{2R(\sin A + \sin B + \sin C)}$$

$$= \frac{4\sin A\sin B\sin C}{2\left(4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}\right)} = 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

**10. (c) :** We have, $\lim\limits_{x\to 0}\frac{f(x)}{x} = 2,$

Given that $f(x) = \min(\sin\sqrt{[m]}x, |x|)$

$$= \lim_{x\to 0}\frac{\min(\sin\sqrt{[m]}x, |x|)}{x} = 2$$

$$= \lim_{x\to 0}\frac{\sin\sqrt{[m]}x}{x} = 2\ \ [\because \sin kx \le |x|\ \forall\ k, x \in R]$$

$$= \lim_{x\to 0}\frac{\sin\sqrt{[m]}x}{\sqrt{[m]}x}\cdot\sqrt{[m]} = 2$$

$\Rightarrow \sqrt{[m]} = 2\ \Rightarrow\ [m] = 4$
$\Rightarrow m \in [4, 5)$ *i.e.* $4 \le m < 5$.

**11. (d) :** $$\int_0^x f(z)dz = x + \int_x^1 zf(z)\,dz \qquad ...(i)$$

Differentiating (i) both sides, we get

$$\frac{d}{dx}\left(\int_0^x f(z)dz\right) = \frac{d}{dx}\left(x + \int_x^1 z\,f(z)dz\right)$$

$\Rightarrow f(x) = 1 + (0 - x\,f(x))$
$\Rightarrow f(x) = 1 - x\,f(x)$

$$\Rightarrow f(x) = \frac{1}{1 + x}$$

$$\therefore\ \int_1^2 f(x)dx = \int_1^2 \frac{1}{1 + x}dx = \log\left(\frac{3}{2}\right)$$

**12. (a) :** We know that $l^2 + m^2 + n^2 = 1$ ... (i)

Since the line makes the same angle $\theta$ with each of $x$ and $z$-axis and $\beta$ with $y$-axis.

$\therefore\ l^2 = \cos^2\theta,\ n^2 = \cos^2\theta$ and $m^2 = \cos^2\beta$

(i) becomes

$2l^2 + m^2 = 1$ or $2n^2 + m^2 = 1$

$\Rightarrow\ 2\cos^2\theta = 1 - \cos^2\beta$

$\Rightarrow\ 2\cos^2\theta = \sin^2\beta$

$\Rightarrow\ 2\cos^2\theta = 3\sin^2\theta \ \Rightarrow\ 5\cos^2\theta = 3$

**13. (b) :** $\displaystyle\int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx = \frac{k}{4}$ ... (i)

$$\Rightarrow \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}+\sqrt{\sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}} = \frac{k}{4}$$

$$\left[\because \int_a^b f(x)dx = \int_a^b f(a+b-x)\,dx\right]$$

$\Rightarrow \displaystyle\int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} = \frac{k}{4}$ ... (ii)

Adding (i) and (ii), we get

$$\int_{\pi/6}^{\pi/3} 1dx = \frac{k}{2} \ \Rightarrow\ \frac{\pi}{6} = \frac{k}{2} \ \Rightarrow\ k = \frac{\pi}{3}$$

**14. (a) :** $\displaystyle\lim_{x\to 0}\frac{\log(1+x^3)}{\sin^3 x} = \lim_{x\to 0}\frac{\log(1+x^3)}{x^3}\cdot\frac{x^3}{\sin^3 x}$

$$= \lim_{x^3\to 0}\frac{\log(1+x^3)}{x^3}\cdot\lim_{x\to 0}\left(\frac{x}{\sin x}\right)^3$$

$$= \lim_{u\to 0}\frac{\log(1+u)}{u}\cdot\lim_{x\to 0}\left(\frac{x}{\sin x}\right)^3$$

$(\because\ x\to 0\ \ \therefore\ x^3\to 0.$ Put $x^3 = u)$

$= 1 \qquad \left[\because\ \displaystyle\lim_{x\to 0}\frac{\sin x}{x} = 1\right]$

**15. (c) :** $\cos^2A + \cos^2(B - A) - 2\cos A\cos B\ \cos(A - B)$

$= \cos^2A + \cos^2(B - A) - \cos(A - B)[\cos(A + B) + \cos(A - B)]$

$= \cos^2A + \cos^2(A - B) - (\cos^2A - \sin^2B) - \cos^2(A - B)$

$= \sin^2B$

**16. (c) :** Here $f(x) = \log_4(\log_5(\log_3(18x - x^2 - 77)))$

The given function is defined when

$\log_5(\log_3(18x - x^2 - 77)) > 0$

$[\because \log x$ is defined $\forall\ x > 0]$

$\Rightarrow\ \log_3(18x - x^2 - 77) > 5^0$

$\Rightarrow\ 18x - x^2 - 77 > 3^1 \ \Rightarrow\ x^2 - 18x + 80 < 0$

$\Rightarrow\ (x - 8)(x - 10) < 0 \ \Rightarrow\ 8 < x < 10$

$\therefore\ x \in (8, 10)$

**17. (c) :** Equation of tangent to parabola $x^2 = 4y$ is

$my + \dfrac{1}{m} = x$, $m$ is arbitrary constant

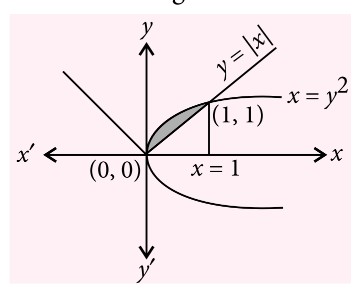
$\therefore\ m\dfrac{dy}{dx} = 1 \ \Rightarrow\ m = \dfrac{1}{dy/dx}$

Now putting the value of $m$ in $my + \dfrac{1}{m} = x$, we get

$$\frac{y}{dy/dx} + \frac{dy}{dx} = x \ \Rightarrow\ \left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$$

$\Rightarrow$ order 1 and degree 2.

**18. (a) :**




The equation of the given curves are

$y^2 = x$ and $y = |x|$

Required area $= \displaystyle\int_0^1 (\sqrt{x} - x)dx$

$$= \left[\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_0^1 = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$$

**19. (b) :** By Lagrange's mean value theorem, we have

$$\frac{f(b)-f(a)}{b-a} = f'(c) \ \Rightarrow\ \frac{f(6)-f(1)}{6-1} \geq 2$$

$\Rightarrow\ f(6) - f(1) \geq 10 \ \Rightarrow\ f(6) + 2 \geq 10$

$\Rightarrow\ f(6) \geq 8 \qquad [\because f(1) = -2]$

**20. (c) :** Given that, $xy = e - e^y$

$$\Rightarrow\ y + x\frac{dy}{dx} = -e^y\frac{dy}{dx}$$

$\Rightarrow\ x\dfrac{dy}{dx} + e^y\dfrac{dy}{dx} + y = 0$ ... (i)

When $x = 0$ then $y = 1$. $\therefore\ \dfrac{dy}{dx} = -\dfrac{1}{e}$

Now, differentiating (i) with respect to $x$, we get

$$e^y \frac{d^2y}{dx^2} + e^y\left(\frac{dy}{dx}\right)^2 + x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$$

Using $x = 0$, $y = 1$, $\frac{dy}{dx} = -\frac{1}{e}$, we get $\frac{d^2y}{dx^2} = \frac{1}{e^2}$

**21. (c) :** $\int \cos\sqrt{x}\,dx$. Let $\sqrt{x} = t \Rightarrow dx = 2t\,dt$

$= 2\int t\cos t\,dt = 2[t\sin t - \int \sin t\,dt]$

$= 2(t\sin t + \cos t)$

$= 2(\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}) + c$

**22. (b):** Given that

$f(x + y) = f(x) + f(y) - xy - 1\ \forall\ x, y \in R$ and $f(1) = 1$

$f(2) = f(1 + 1) = f(1) + f(1) - 1 - 1 = 0$

$f(3) = f(2 + 1) = f(2) + f(1) - 2\cdot 1 - 1 = -2$

$f(n + 1) = f(n) + f(1) - n - 1 = f(n) - n$

$f(n + 1) < f(n)$

Thus, $f(1) > f(2) > f(3) > ....$ and $f(1) = 1$

$\therefore\ f(1) = 1$ and $f(n) < 1$, for $n > 1$

Hence $f(n) = n$, $n \in n$ has only one solution $n = 1$.

**23. (d):** Let the G.P. be $a, ar, ar^2, ....., ar^{n-1}$

Given that, $t_n = t_{n+1} + t_{n+2}$

$\therefore\ a = ar + ar^2$

$\Rightarrow r^2 + r = 1 \Rightarrow r^2 + r - 1 = 0$

$$\Rightarrow r = \frac{-1 \pm \sqrt{1^2 - 4\cdot 1\cdot(-1)}}{2\cdot 1} = \frac{-1 \pm \sqrt{5}}{2}$$

$$\Rightarrow r = \frac{\sqrt{5} - 1}{2}$$

$$\left[\because r = \frac{-1 - \sqrt{5}}{2} \text{ is rejected as terms are positive}\right]$$

$$\Rightarrow r = 2\left(\frac{\sqrt{5} - 1}{4}\right) = 2\sin 18°$$

**24. (c) :** Given that $\log_{10}(x^2 - 2x - 2) \le 0$

$\Rightarrow x^2 - 2x - 2 \le 10^0 \Rightarrow x^2 - 2x - 3 \le 0$

$\Rightarrow -1 \le x \le 3$ ... (i)

For logarithm to be defined, $x^2 - 2x - 2 > 0$

$\Rightarrow x > 1 + \sqrt{3}$ and $x < 1 - \sqrt{3}$ ... (ii)

From (i) and (ii), common values of $x$ are given by

$-1 \le x < 1 - \sqrt{3}$ or $1 + \sqrt{3} < x \le 3$

**25. (d):** Let $A$ and $B$ denote the set of employees who like bananas and apples respectively. Further, let the total number of employees be 100.

$\therefore\ n(A) = 70, n(B) = 64, n(U) = 100$ and $n(A \cap B) = x$

Now $n(A \cup B) \le 100$

$\therefore\ n(A) + n(B) - n(A \cap B) \le 100$

$70 + 64 - x \le 100$

$\Rightarrow x \ge 34$ ... (i)

Again $A \cap B \subseteq B$. $\therefore\ n(A \cap B) \le n(B)$

$\Rightarrow x \le 64$ ... (ii)

From (i) and (ii), we get

$34 \le x \le 64$

**26. (c) :** Here $A = \{2, 3, 4, 5\}$ and $B = \{3, 6, 7, 10\}$

Given that $(x, y) \in R \Leftrightarrow x$ divides $y$

$\therefore\ (2, 6) \in R, (2, 10) \in R, (3, 3) \in R, (3, 6) \in R, (5, 10) \in R$

$\Rightarrow R = \{(2, 6), (2, 10), (3, 3), (3, 6), (5, 10)\}$

$\therefore\ R^{-1} = \{(6, 2), (10, 2), (3, 3), (6, 3), (10, 5)\}$

**27. (a) :** Let $a$ and $b$ are two numbers.

$\therefore\ a, A_1, A_2, b$ are in A.P.

$\Rightarrow A_1 + A_2 = a + b$

$a, G_1, G_2, b$ are in G.P. $\Rightarrow G_1G_2 = ab$

Again, $a, H_1, H_2, b$ are in G.P.

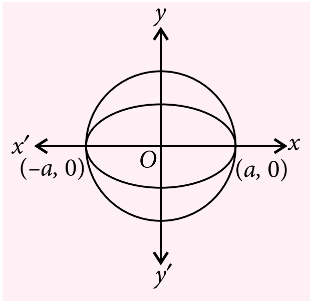
$$\therefore \frac{1}{H_1} + \frac{1}{H_2} = \frac{1}{a} + \frac{1}{b}$$

$$\Rightarrow \frac{H_1 + H_2}{H_1H_2} = \frac{a + b}{ab} = \frac{A_1 + A_2}{G_1G_2}$$

$$\Rightarrow \frac{A_1 + A_2}{H_1 + H_2} = \frac{G_1G_2}{H_1H_2}$$

**28. (b):** Four questions with correct answers can be chosen in $^6C_4$ ways. Since only one of the four alternatives is correct, the wrong answers can be given in 3 different ways for each of the two remaining questions attempted unsuccessfully by the candidate. The desired number of ways $= {^6C_4}\cdot 3^2 = 135$.

**29. (d):** Let $a$ is the radius of the circle.




$\therefore$ The length of the major axis of the ellipse is $2a$.

Area of the region between circle and ellipse is

$$\pi a^2 - \pi ab = \pi a^2 - \pi a^2\frac{b}{a} = \pi a^2 - \pi a^2\sqrt{\frac{b^2}{a^2}}$$

$$= \pi a^2 - \pi a^2\sqrt{1 - \left(1 - \frac{b^2}{a^2}\right)}$$

$$= \pi a^2 - \pi a^2\sqrt{1-e^2} \quad \left[\because\ e = \sqrt{1-\frac{b^2}{a^2}}\right]$$

$\therefore$ Probability that the randomnly chosen point in the circle will fall outside of the ellipse is

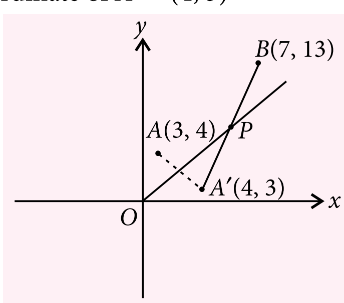
$$\text{Probability} = \frac{\pi a^2 - \pi a^2\sqrt{1-e^2}}{\pi a^2}$$

$$= 1-\sqrt{1-e^2} = 1-\sqrt{1-\left(\frac{2\sqrt{2}}{3}\right)^2}$$

$$= 1-\sqrt{1-\frac{8}{9}} = 1-\frac{1}{3} = \frac{2}{3}$$

**30. (d):** Let $A'$ be the image of $A$ in $y = x$ ... (i)

$\therefore$ Co-ordinate of $A' \equiv (4, 3)$




Now $PA' + PB$ is minimum if $PA'B$ are collinear

$\therefore$ equation of $PA'B$ is

$$y-3 = \frac{13-3}{7-4}(x-4) \Rightarrow y-3 = \frac{10}{3}(x-4) \text{ ... (ii)}$$

Solving (i) and (ii), we get the point $P$ whose co-ordinate is $\left(\frac{31}{7}, \frac{31}{7}\right)$

**31. (b):** The given equation is

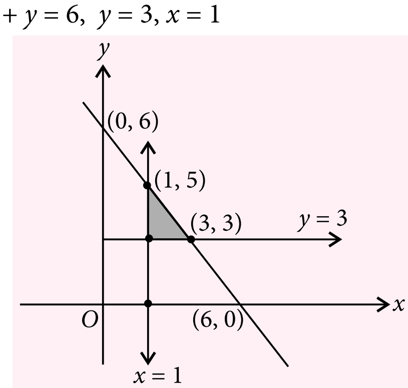
$(x + y - 6)(xy - 3x - y + 3) = 0$

$\therefore$ $x + y - 6 = 0$

or $xy - 3x - y + 3 = 0$ or $(x - 1)(y - 3) = 0$

So, the equations of the sides of the triangle are

$x + y = 6,\ y = 3, x = 1$




The shaded triangle is right angled at (1, 3).

$\therefore$ The circumcircle is the circle on (3, 3) and (1, 5) as ends of a diameter.

$\therefore$ Equation of the circle whose extremities of the diameter (3, 3) and (1, 5) is

$(x - 3)(x - 1) + (y - 3)(y - 5) = 0$

*i.e.* $x^2 + y^2 - 4x - 8y + 18 = 0$

**32. (b):** Let $\alpha$, $\beta$ are the roots of the equation $x^2 + ax + b = 0$ and that of $x^2 + bx + a = 0$ be $\gamma$ and $\delta$.

$\therefore$ $\alpha + \beta = -a$ and $\alpha\beta = b$

$\gamma + \delta = -b$ and $\gamma\delta = a$

We also have $\alpha - \beta = \gamma - \delta$

$\Rightarrow$ $(\alpha - \beta)^2 = (\gamma - \delta)^2$

$\Rightarrow$ $(\alpha + \beta)^2 - 4\alpha\beta = (\gamma + \delta)^2 - 4\gamma\delta$

$\Rightarrow$ $a^2 - 4b = b^2 - 4a$

$\Rightarrow$ $a^2 - b^2 + 4(a - b) = 0$

$\Rightarrow$ $(a + b)(a - b) + 4(a - b) = 0$

$\Rightarrow$ $(a - b)(a + b + 4) = 0$

$\because$ $a \neq b$ $\therefore$ $a + b + 4 = 0$ *i.e.* $a + b = -4$.

**33. (a):** $\Delta = \begin{vmatrix} \cos\frac{\theta}{2} & 1 & 1 \\ 1 & \cos\frac{\theta}{2} & -\cos\frac{\theta}{2} \\ -\cos\frac{\theta}{2} & 1 & -1 \end{vmatrix}$

$$= \cos\frac{\theta}{2}(0) - 1\left(-1-\cos^2\frac{\theta}{2}\right) + 1\left(1+\cos^2\frac{\theta}{2}\right)$$

$$\Delta = 2\cos^2\frac{\theta}{2} + 2$$

Now, $\Delta_{max} = 2 + 2 = 4$

and $\Delta_{min} = 2 \times 0 + 2 = 2$ $\quad \left[\because\ 0 \le \cos^2\frac{\theta}{2} \le 1\right]$

$\therefore$ $\Delta \in [2, 4]$

**34. (a):** The given system of equations is

$\alpha x - 3y + z = 0,\ x + \alpha y + 3z = 1,\ 3x + y + 5z = 2$

Now, $\Delta = \begin{vmatrix} \alpha & -3 & 1 \\ 1 & \alpha & 3 \\ 3 & 1 & 5 \end{vmatrix}$

$= \alpha(5\alpha - 3) + 3(5 - 9) + 1(1 - 3\alpha)$

$= 5\alpha^2 - 6\alpha - 11 = (5\alpha - 11)(\alpha + 1)$

The system does not have a unique solution iff $\Delta = 0$

$\Rightarrow$ $\alpha = \frac{11}{5}$ or $-1$

**35. (b):** The given line is parallel to the vector

$$\vec{n} = \hat{i} - \hat{j} + 2\hat{k}$$

Given that, the required plane passes through the point (2, 3, 1) *i.e.* $2\hat{i} + 3\hat{j} + \hat{k}$

Also given that the plane is perpendicular to $\vec{n} = \hat{i} - \hat{j} + 2\hat{k}$

So, its equation is $[(\vec{r} - (2\hat{i} + 3\hat{j} + \hat{k})] \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 0$

$\Rightarrow \quad \vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 1$

**36. (c):** $|\vec{a} + \vec{b} + \vec{c}|^2 = 6$

*i.e.* $\quad (\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = 6$

$\Rightarrow \quad |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 6$

Given that $|\vec{a}| = |\vec{b}| = |\vec{c}|$ and angle between each pair is $\pi/3$.

$\therefore \quad \vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\frac{\pi}{3} = \frac{1}{2}|\vec{a}|^2$ and

$\vec{b} \cdot \vec{c} = \frac{1}{2}|\vec{a}|^2 = \vec{c} \cdot \vec{a}$

$\Rightarrow \quad 3|\vec{a}|^2 + 2\left(\frac{1}{2}|\vec{a}|^2 + \frac{1}{2}|\vec{a}|^2 + \frac{1}{2}|\vec{a}|^2\right) = 6$

$\Rightarrow \quad 3|\vec{a}|^2 + 2 \times \frac{3}{2}|\vec{a}|^2 = 6,$

$\Rightarrow \quad 6|\vec{a}|^2 = 6. \quad \therefore \quad |\vec{a}|^2 = 1 \Rightarrow |\vec{a}| = 1$

**37. (c):** The equation of the parabola is

$y^2 = 8x = 4 \cdot 2x$

$\therefore$ Focus (2, 0)

Let the point on the parabola be $(2t^2, 4t)$

$\therefore \quad \sqrt{(2t^2 - 2)^2 + (4t)^2} = 4$

$\Rightarrow \quad 4(t^2 - 1)^2 + 16t^2 = 16$

$\Rightarrow \quad (t^2 - 1)^2 + 4t^2 = 4$

$\Rightarrow \quad (t^2 + 1)^2 = 4 \quad \Rightarrow \quad t^2 + 1 = \pm 2$

$\therefore \quad t^2 = 2 - 1 = 1 \qquad [\because t^2 \neq -3]$

$\therefore \quad t = \pm 1$

$\therefore$ The required points are (2, 4) and (2, –4).

**38. (c):** Here $f(x) = \cos^{-1}\left\{\frac{1}{\sqrt{13}}(2\cos x - 3\sin x)\right\}$

$+ \sin^{-1}\left\{\frac{1}{\sqrt{13}}(2\sin x + 3\cos x)\right\}$

Let $\frac{2}{\sqrt{13}} = \cos\alpha$ and $\frac{3}{\sqrt{13}} = \sin\alpha$

$\therefore \quad f(x) = \cos^{-1}(\cos x \cos\alpha - \sin x \sin\alpha)$

$+ \sin^{-1}(\sin x \cos\alpha + \cos x \sin\alpha)$

$= \cos^{-1}\cos(x + \alpha) + \sin^{-1}\sin(x + \alpha)$

$f(x) = 2x + 2\alpha \quad \because \frac{d}{dx}\{f(x)\} = 2$

Let $g(x) = \sqrt{1 + x^2} \quad \therefore \quad \frac{d}{dx}\{g(x)\} = \frac{1}{2}\frac{2x}{\sqrt{1 + x^2}}$

$\therefore \quad \frac{df(x)}{dg(x)} = \frac{2}{\frac{x}{\sqrt{1+x^2}}} = \frac{2\sqrt{1+x^2}}{x}$

**39. (c):** $\lim_{x \to 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2} \quad \left[\text{form } \frac{0}{0}\right]$

$= \lim_{x \to 0} \frac{2f'(x) - 6f'(2x) + 4f'(4x)}{2x}$

[By L-Hospital's rule]

$= \lim_{x \to 0} \frac{f'(x) - 3f'(2x) + 2f'(4x)}{x} \quad \left[\frac{0}{0}\right]$

$= \lim_{x \to 0} \frac{f''(x) - 6f''(2x) + 8f''(4x)}{1}$

$= f''(0) - 6f''(0) + 8f''(0) = 3f''(0) \qquad [\because f''(0) = 4]$

$= 3 \times 4 = 12$

**40. (a):** Given that $b_1 + b_4 + b_7 + ... + b_{25} + b_{28} = 220$

$\because \quad$ 1, 4, 7, 10, ..., 25, 28 are in A.P.

$\therefore \quad 1 + (n - 1)3 = 28$

$\Rightarrow \quad 3n = 30 \quad$ *i.e.* $\quad n = 10$

and $b_1 + b_{28} = b_4 + b_{25} = b_7 + b_{22} = .... = b_{13} + b_{16}$

$\therefore \quad 5(b_1 + b_{28}) = 220 \quad \Rightarrow \quad b_1 + b_{28} = 44$

Now, $b_1 + b_2 + b_3 + .... + b_{28}$

$= 14(b_1 + b_{28}) = 14 \times 44 = 616$

**41. (c):** $(1 - 3x + 3x^2 - x^3)^{2n} = (1 - x)^{6n}$

Since the power of the expansion is even, so there will be a unique middle term, which is

$t_{(3n+1)} = {}^{6n}C_{3n}(-x)^{3n}$

$= \frac{6n!}{3n!\,3n!}(-x)^{3n}$

**42. (d):** $(x^2 + y^2)dx - 2xy\,dy = 0$

$\Rightarrow \quad 2xydy = (x^2 + y^2)dx$

$\Rightarrow \quad \frac{dy}{dx} = \frac{x^2 + y^2}{2xy} \qquad$ [Putting $y = vx$]

$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{1 + v^2}{2v} \quad \Rightarrow \quad x\frac{dv}{dx} = \frac{1 + v^2}{2v} - v$

$\Rightarrow \quad x\frac{dv}{dx} = \frac{1 - v^2}{2v} \quad \Rightarrow \quad \int \frac{2v}{1 - v^2}dv = \int \frac{dx}{x}$

$\Rightarrow \quad \log(1 - v^2) = -\log x + \log c$

$$\Rightarrow\ 1-v^2=\frac{c}{x}\ \Rightarrow\ \frac{x^2-y^2}{x^2}=\frac{c}{x}$$

$$\Rightarrow\ \frac{x^2-y^2}{x}=c$$

**43. (d):** Vector of unit length perpendicular to given vectors = $\pm\left\{\frac{\vec{a}\times\vec{b}}{|\vec{a}||\vec{b}|}\right\}$

Hence 2 such vectors.

**44. (b):** The equation of the hyperbola is

$$x^2-y^2\sec^2\theta=4\ \Rightarrow\ \frac{x^2}{4}-\frac{y^2}{4\cos^2\theta}=1$$

The equation of the ellipse is

$$x^2\sec^2\theta+y^2=16\ \Rightarrow\ \frac{x^2}{16\cos^2\theta}+\frac{y^2}{16}=1$$

Now, by the problem,

$$1+\frac{4\cos^2\theta}{4}=3\left(1-\frac{16\cos^2\theta}{16}\right)$$

$$\Rightarrow\ \frac{4+4\cos^2\theta}{4}=3\left(\frac{16-16\cos^2\theta}{16}\right)$$

$$\Rightarrow\ 1+\cos^2\theta=3(1-\cos^2\theta)$$

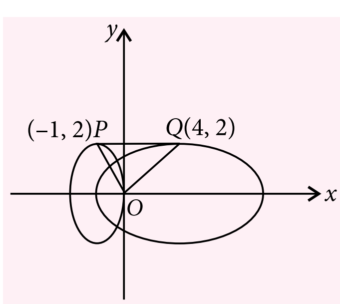
$$\Rightarrow\ 4\cos^2\theta=2\ \Rightarrow\ \cos\theta=\pm\frac{1}{\sqrt{2}}$$

$$\therefore\ \theta=\frac{\pi}{4},\ \frac{3\pi}{4}$$

**45. (c) :** The given equations of the ellipse are

$$4(x-4)^2+25y^2=100\ \text{ or }\ \frac{(x-4)^2}{25}+\frac{y^2}{4}=1$$

and $4(x+1)^2+y^2=4$ or $\frac{(x+1)^2}{1}+\frac{y^2}{4}=1$




Slope of $OP$ × slope of $OQ$ $=\frac{2}{-1}\times\frac{2}{4}=-1$

$\therefore$ $OP$ and $OQ$ are perpendicular to each other.

**46. (c) :** Given that $\frac{5z_2}{11z_1}$ is purely imaginary.

$$\therefore\ \frac{5z_2}{11z_1}=i\lambda\ \Rightarrow\ \frac{z_2}{z_1}=\frac{11}{5}i\lambda$$

$$\text{Now, }\left|\frac{2z_1+3z_2}{2z_1-3z_2}\right|=\left|\frac{1+\frac{3}{2}\cdot\frac{z_2}{z_1}}{1-\frac{3}{2}\cdot\frac{z_2}{z_1}}\right|=\left|\frac{1+\frac{33}{10}i\lambda}{1-\frac{33}{10}i\lambda}\right|$$

$= 1$ [$\because |a+ib|=|a-ib|$]

**47. (b):** Given that $\sum_{i=1}^{m}a_{ii}=0$

$\Rightarrow\ x-5+x^2-10+x-7-2=0$

$\Rightarrow\ x^2+2x-24=0\ \Rightarrow\ (x-4)(x+6)=0$

$\Rightarrow\ x=-6,4$

Note that : Trace of a matrix $A=(a_{ij})_{m\times m}$ is $\sum_{i=1}^{m}a_{ii}$

**48. (d):** Given that $f(x)=\phi(x)+f(1-x)$ and $f''(x)<0$ in $0\le x\le 1$

$\therefore\ \phi'(x)=f'(x)-f'(1-x)$

$\because\ f''(x)<0$ in $0\le x\le 1$

$\Rightarrow\ f'(x)$ decreases in $0<x<1$.

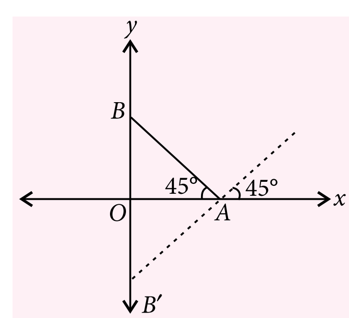
Now, if $x>1-x$ i.e. $x>\frac{1}{2}$

$\phi'(x)<0$ in $\frac{1}{2}<x<1$ $\Rightarrow$ $\phi(x)$ decreases in $\frac{1}{2}<x<1$

and if $x<1-x$ i.e. $x<\frac{1}{2}$. $\phi'(x)>0$

$\Rightarrow\ \phi(x)$ increases in $0<x<\frac{1}{2}$

**49. (a):** Since the tangent is equally inclined to the axes, it must be either the line $AB$ or $B'A$ as shown in the diagram.




$\therefore$ Slope = ±1

$$i.e.\ \frac{dy}{dx}=-\left(\frac{y}{x}\right)^{1/3}=\pm1$$

$$\Rightarrow\ y=\pm x=\pm\frac{a}{2\sqrt{2}}$$

$\therefore$ There are four points of contact, one each in the 4 quadrant

**50. (c) :** Here, probability that $A$ will solve the problem is 2/5.

Probability that $B$ will solve the problem is 2/3.

Problem will be solved if atleast any one can solve the problem.

*i.e.* $P(A \cup B)$ is the required probability.

Now, $P(A \cup B) = 1 - P(A \cup B)^C$

$= 1 - P(A^C \cap B^C)$

$= 1 - P(A^C) \cdot P(B^C)$

[$\because$ The two events are independent]

$$= 1 - \frac{3}{5} \times \frac{1}{3} = 1 - \frac{1}{5} = \frac{4}{5}$$

**51. (a) :** Let $X$ denotes the number of tails in tossing 6 coins.

Here, $X \sim B\left(6, \frac{1}{2}\right)$

$$\therefore \quad P(X = x) = {}^6C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{6-x} = {}^6C_x \left(\frac{1}{2}\right)^6$$

$$\therefore \quad P(X = 0) = {}^6C_0 \cdot \frac{1}{2^6} = 1 \times \frac{1}{2^6} = \frac{1}{64}$$

We want $P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0)$

$$= 1 - \frac{1}{64} = \frac{63}{64}$$

**52. (a) :** Given that

$$P(A) = \frac{1}{4},\ P(A \mid B) = \frac{1}{2},\ P(B \mid A) = \frac{2}{3}$$

Now, $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

$$\Rightarrow \quad P(B) = \frac{P(A \cap B)}{P(A \mid B)} = 2P(A \cap B)$$

Again, $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} \Rightarrow P(A) = \dfrac{P(A \cap B)}{P(B \mid A)}$

$$\therefore \ P(A) = \frac{3}{2} P(A \cap B) \ \Rightarrow \ \frac{1}{4} \times \frac{2}{3} = P(A \cap B)$$

$$\Rightarrow \quad P(A \cap B) = \frac{1}{6}$$

Now, $P(B) = 2 \times \dfrac{1}{6} = \dfrac{1}{3}$

**53. (b):** $P(n) = 2^n > n \quad \Rightarrow \quad P(k) = 2^k > k$

$\Rightarrow \quad 2 \cdot 2^k > 2k \ \Rightarrow \ 2 \cdot 2^k > 2k > k + 1$ as $k \geq 2$

**54. (d):** $\because -|z| \in R,\ \therefore\ \arg(-|z|) = \pi$

$$\text{Now, } \int_0^{50} \arg(-|z|)\, dx = \int_0^{50} \pi dx = \pi [x]_0^{50} = 50\pi$$

**55. (b):** The given equation is

$3\cos 2\theta + 13\sin\theta - 8 = 0$

$\Rightarrow \quad 3(1 - 2\sin^2\theta) + 13\sin\theta - 8 = 0$

$\Rightarrow \quad 3 - 6\sin^2\theta + 13\sin\theta - 8 = 0$

$\Rightarrow \quad 6\sin^2\theta - 13\sin\theta + 5 = 0$

$\Rightarrow \quad 6\sin^2\theta - 10\sin\theta - 3\sin\theta + 5 = 0$

$\Rightarrow \quad 2\sin\theta(3\sin\theta - 5) - 1(3\sin\theta - 5) = 0$

$\Rightarrow \quad (3\sin\theta - 5)(2\sin\theta - 1) = 0$

$$\because \ \sin\theta \neq \frac{5}{3}. \quad \therefore \ \sin\theta = \frac{1}{2} \text{ gives } \theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

**56. (d):** The relation $\{(b, x), (a, y), (b, z), (c, z)\}$ has two ordered pairs $(b, x)$ and $(b, z)$ whose first elements are same. Hence it is not a mapping.

**57. (a) :** We know, for moderately asymmetrical distribution,

mode = 3 median – 2 mean

= 3(22) – 2(18) = 66 – 36 = 30

**58. (c) :** We are given $\vec{P} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{Q} = \hat{i} + 3\hat{k}$

Now, $[\vec{R}\ \vec{P}\ \vec{Q}] = |\vec{R}||\vec{P} \times \vec{Q}|\cos\phi$ where $\vec{R}$ is unit vector.

$\phi$ is angle between $\vec{R}$ and $\vec{P} \times \vec{Q}$

$\Rightarrow \quad |\vec{P} \times \vec{Q}|\cos\phi \leq |\vec{P} \times \vec{Q}|$

$\therefore \quad$ Minimum value of $[\vec{R}\,\vec{P}\,\vec{Q}] = |\vec{P} \times Q| = \sqrt{59}$

$$\left[\begin{array}{l} \because \ \vec{P} \times \vec{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 0 & 3 \end{vmatrix} = \hat{i}(3) - \hat{j}(7) + \hat{k}(-1) \\ \therefore \ |\vec{P} \times \vec{Q}| = \sqrt{3^2 + 7^2 + (-1)^2} = \sqrt{59} \end{array}\right]$$

**59. (d):** Given that C.V. = 60, S.D. = 20

$$\text{But C.V.} = \frac{\text{S.D.}}{\text{Mean}} \times 100$$

$$\therefore \ 60 = \frac{20}{\text{Mean}} \times 100. \ \therefore \ \text{Mean} = 33.33 \text{ (nearly)}$$

**60. (a) :** $\tan\left\{\left(\omega^{200} + \dfrac{1}{\omega^{200}}\right)\pi + \dfrac{\pi}{4}\right\}$,

$\omega$ is a cube root of unity

$$= \tan\left[\left\{(\omega^3)^{66} \cdot \omega^2 + \frac{1}{(\omega^3)^{66} \cdot \omega^2}\right\}\pi + \frac{\pi}{4}\right]$$

$$= \tan\left[\left(\omega^2 + \frac{1}{\omega^2}\right)\pi + \frac{\pi}{4}\right] \quad [\because \ \omega^3 = 1]$$

$$= \tan\left[(\omega^2 + \omega)\,\pi + \frac{\pi}{4}\right]$$

$= \tan\left(-\pi + \frac{\pi}{4}\right)$ [$\because 1 + \omega + \omega^2 = 0$]

$= -\tan\left(\pi - \frac{\pi}{4}\right)$ [$\because \tan(-\theta) = -\tan\theta$]

$= \tan\frac{\pi}{4}$ [$\because \tan(\pi - \theta) = -\tan\theta$]

$= 1$

**61. (d):** Given that $l, G_1, G_2, G_3, n$ are in G.P.
Let the common ratio be $r$.

$\therefore\ G_1 = lr, G_2 = lr^2, G_3 = lr^3, n = lr^4$

Now, $G_1^4 + 2G_2^4 + G_3^4 = (lr)^4 + 2(lr^2)^4 + (lr^3)^4$

$= l^4r^4 + 2l^4r^8 + l^4r^{12}$

$= l^3(lr^4) + 2l^2(lr^4)^2 + l(lr^4)^3$

$= l^3n + 2l^2n^2 + ln^3$

$= ln(l + 2nl + n^2) = ln(l + n)^2$

$= 4m^2nl$ $\left[\because \frac{l+n}{2} = m\right]$

**62. (a):** Given that $\alpha$ and $\beta$ are the two roots of the equation $x^2 - 6x - 2 = 0$.

$\therefore\ \alpha^2 - 6\alpha - 2 = 0$ and $\beta^2 - 6\beta - 2 = 0$

Multiplying the above two equations by $\alpha^n$ and $\beta^n$ respectively, we get

$\alpha^{n+2} - 6\alpha^{n+1} - 2 = 0$

and $\beta^{n+2} - 6\beta^{n+1} - 2 = 0$

Subtracting, we get

$(\alpha^{n+2} - \beta^{n+2}) - 6(\alpha^{n+1} - \beta^{n+1}) - 2(\alpha^n - \beta^n) = 0$

*i.e.* $a_{n+2} - 6a_{n+1} - 2a_n = 0$ [$\because a_n = \alpha^n - \beta^n$]

$\Rightarrow a_{n+2} - 2a_n = 6a_{n+1}$

$\Rightarrow \frac{a_{n+2} - 2a_n}{2a_{n+1}} = 3$

Put $n = 8$ to get $\frac{a_{10} - 2a_8}{2a_9} = 3$

**63. (c):** *Reflexive property* : For all $(a, b) \in N \times N$, $(a, b)\ R\ (a, b)$ since $a + b = b + a$. Hence

$(a, b)\ R\ (a, b)\ \forall\ (a, b) \in N \times N$

$\therefore$ $R$ is reflexive.

*Symmetric property :*

If $(a, b)\ R\ (c, d)$ then $a + d = b + c$.

Now, when $a + d = b + c$ then $c + b = d + a$. Hence $(c, d)\ R\ (a, b)$.

Hence, if $(a, b)\ R\ (c, d)$ then $(c, d)\ R\ (a, b)\ \forall$ all $(a, b), (c, d) \in N \times N$. $\therefore$ $R$ is symmetric.

*Transitive property:*

Let $(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f)$.

Then $a + d = b + c$ and $c + f = d + e$

$\therefore\ a + d + c + f = b + c + d + e$

*i.e.* $a + f = b + e$. Hence $(a, b)\ R\ (e, f)$

Hence if $(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f)$ then $(a, b)\ R\ (e, f)\ \forall\ (a, b), (c, d), (e, f) \in N \times N$

$\therefore$ $R$ is transitive.

Since $R$ is reflexive, symmetric and transitive, hence $R$ is an equivalence relation on $N \times N$.

**64. (c):** Let us define the events first.

$E_1$ = the examinee guesses the answer

$E_2$ = the examinee copies the answer

$E_3$ = the examinee knows the answer

and $A$ = the examinee has answered the question correctly.

$\therefore\ P(E_1) = \frac{1}{3},\ P(E_2) = \frac{1}{6}$ and $P(E_3) = 1 - \frac{1}{3} - \frac{1}{6} = \frac{1}{2}$

[$\because E_1, E_2, E_3$ are mutually exclusive and exhaustive events, therefore $P(E_1) + P(E_2) + P(E_3) = 1$]

Now, $P(A|E_1) = \frac{1}{4},\ P(A|E_2) = \frac{1}{8},\ P(A|E_3) = 1$

Now,

$$P(E_3|A) = \frac{P(E_3)P(A|E_3)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3)}$$

$$= \frac{\frac{1}{2}\cdot 1}{\frac{1}{3}\cdot\frac{1}{4} + \frac{1}{6}\cdot\frac{1}{8} + \frac{1}{2}\cdot 1} = \frac{24}{29}$$

**65. (c):** $\tan^{-1} y = \tan^{-1} x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$= \tan^{-1}x + 2\tan^{-1}x = 3\tan^{-1}x$

$\tan^{-1} y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$ $\left[\because |x| < \frac{1}{\sqrt{3}}\right]$

$\therefore\ y = \frac{3x - x^3}{1 - 3x^2}$

**66. (c):** Equation of the line passing through the point of intersection of $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{b} + \frac{y}{a} = 1$ is

$$\left(\frac{x}{a} + \frac{y}{b} - 1\right) + \lambda\left(\frac{x}{b} + \frac{y}{a} - 1\right) = 0 \quad \text{... (i)}$$

Since (i) is passing through (0, 0)

$\therefore\ -1 + \lambda(-1) = 0 \Rightarrow \lambda = -1$

$\therefore$ (i) becomes

$$\frac{x}{a} + \frac{y}{b} - 1 - 1\left(\frac{x}{b} + \frac{y}{a} - 1\right) = 0$$

$$\Rightarrow \left(\frac{1}{a}-\frac{1}{b}\right)x+\left(\frac{1}{b}-\frac{1}{a}\right)y=0$$

$\Rightarrow \quad x - y = 0 \quad \Rightarrow \quad y - x = 0$

**67. (d):** Here $[\pi^2 x]$ is an integer $\forall\, x \in R$ and so $2\pi[\pi^2 x]$ is an integral multiple of $\pi$.

Thus, $\sin(2\pi[\pi^2 x]) = 0$ and $5 + [x^2] \neq 0\ \forall\, x \in R$.

$\therefore \quad f(x) = 0\ \forall\, x \in R$

Thus, $f(x)$ is a constant function and so it is continuous and differentiable any number of times $\forall\, x \in R$.

**68. (b):** $\displaystyle\int \frac{dx}{x^{22}(x^7-6)} = \int \frac{dx}{x^{29}\left(1-\frac{6}{x^7}\right)}$

$$\left[\text{Let } 1-\frac{6}{x^7}=z \Rightarrow \frac{42}{x^8}dx=dz \text{ and } x^7=\frac{6}{1-z}\right]$$

$$=\frac{1}{42}\int\frac{(1-z)^3 dz}{(6^3)z}=\frac{1}{(42)\cdot(216)}\int\frac{(1-3z+3z^2-z^3)}{z}dz$$

$$=\frac{1}{54432}[\ln z^6+9z^2-2z^3-18z]+c$$

**69. (d)**

**70. (a):** We have, $f(x) = px + qy$

$$f(x)=px+q\frac{r^2}{x} \quad [\because xy = r^2]$$

$$\therefore \quad f'(x)=p-\frac{qr^2}{x^2}$$

$$f''(x)=\frac{2qr^2}{x^3}$$

Now, $f'(x) = 0$ gives $x=\pm r\sqrt{\frac{q}{p}}$

and $f''(x)>0$ for $x=r\sqrt{\frac{q}{p}}$

$$\therefore \quad f(x)_{\min}=pr\sqrt{\frac{q}{p}}+\frac{qr^2}{r\sqrt{\frac{q}{p}}}=\sqrt{pq}\cdot r+\sqrt{pq}\cdot r$$

$$=2r\sqrt{pq}$$

**71. (a, b, c, d):** $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})$ = scalar triple product of $\vec{a},\vec{b},\vec{c}\times\vec{d}$ $=\vec{a}\cdot\{\vec{b}\times(\vec{c}\times\vec{d})\}$

$(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})$ = scalar triple product of $\vec{a}\times\vec{b},\ \vec{c},\ \vec{d}$

$=\{(\vec{a}\times b)\times\vec{c})\}\cdot\vec{d}$

$$(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})=\vec{a}\cdot\{\vec{b}\times(\vec{c}\times\vec{d})\}$$

$$=\vec{a}\cdot\{(\vec{b}\cdot\vec{d})\vec{c}-(\vec{b}\cdot\vec{c})\vec{d}\}$$

$$=(\vec{b}\cdot\vec{d})(\vec{a}\cdot\vec{c})-(\vec{b}\cdot\vec{c})(\vec{a}\cdot\vec{d})$$

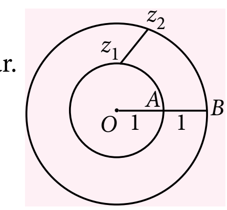
**72. (a, b, c):** $|2z_1 + z_2| \le |2z_1| + |z_2| \le 2|z_1| + |z_2|$

$\le 2 \times 1 + 2 \le 4$

From the figure, $|z_1 - z_2|$ is least when $O, z_1, z_2$ are collinear.

Then $|z_1 - z_2| = 1$

Again,




$$\left|z_2+\frac{1}{z_1}\right|\le|z_2|+\frac{1}{|z_1|}=2+\frac{1}{1}=3$$

**73. (a, b):** $\cos\frac{A}{2}=\sqrt{\frac{b+c}{2c}}$

$$\Rightarrow \quad \cos^2\frac{A}{2}=\frac{b+c}{2c} \quad \Rightarrow \quad 2\cos^2\frac{A}{2}=\frac{b+c}{c}$$

$$\Rightarrow \quad 1+\cos A=\frac{b+c}{c} \quad \Rightarrow \quad \cos A=\frac{b+c}{c}-1=\frac{b}{c}$$

$$\therefore \quad \sin A=\sqrt{1-\cos^2 A}=\sqrt{1-\frac{b^2}{c^2}}=\frac{\sqrt{c^2-b^2}}{c}=\frac{a}{c}$$

Now, $\Delta=\frac{1}{2}bc\sin A=\frac{1}{2}bc\times\frac{a}{c}=\frac{1}{2}ab$

Again, $2R=\frac{a}{\sin A}=\frac{a}{a/c}=c$

$\therefore \quad R=\frac{1}{2}c$

**74. (a, b):** $(x + y + z)^{25} = \{x + (y + z)\}^{25}$

$= {}^{25}C_0\, x^{25} + {}^{25}C_1\, x^{24}(y + z) + \ldots\ldots$

$+ {}^{25}C_r\, x^{25-r}\,(y + z)^r + \ldots$

$= \ldots. + {}^{25}C_r\, x^{25-r}\,(\ldots. + {}^{r}C_k\, y^{r-k}\, z^k + \ldots) + \ldots.$

$\because 8 + 9 + 9 \neq 25$, so there is no term like $x^8 y^9 z^9$.

The number of terms $= 1 + 2 + 3 + \ldots + 26$

$$=\frac{26\times 27}{2}=13\times 27=351$$

**75. (a, b, d):** $\Delta=\begin{vmatrix} x^2 & y^2+z^2 & yz \\ y^2 & z^2+x^2 & zx \\ z^2 & x^2+y^2 & xy \end{vmatrix}$

$$=\begin{vmatrix} x^2+y^2+z^2 & y^2+z^2 & yz \\ x^2+y^2+z^2 & z^2+x^2 & zx \\ x^2+y^2+z^2 & x^2+y^2 & xy \end{vmatrix} [C_1' = C_1 + C_2]$$

$$=(x^2+y^2+z^2)\begin{vmatrix} 1 & y^2+z^2 & yz \\ 1 & z^2+x^2 & zx \\ 1 & x^2+y^2 & xy \end{vmatrix}$$

$$=(x^2+y^2+z^2)\begin{vmatrix} 0 & y^2-x^2 & z(y-x) \\ 0 & z^2-y^2 & x(z-y) \\ 1 & x^2+y^2 & xy \end{vmatrix}$$

$[R_1' = R_1 - R_2 \text{ and } R_2' = R_2 - R_3]$

$$=(x^2+y^2+z^2)(y-x)(z-y)\begin{vmatrix} 0 & y+x & z \\ 0 & z+y & x \\ 1 & x^2+y^2 & xy \end{vmatrix}$$

$= (x^2 + y^2 + z^2)(y - x)(z - y)[1(xy + x^2 - z^2 - yz)]$
$= (x^2 + y^2 + z^2)(y - x)(z - y)(x - z)(x + y + z)$

**76. (a, c) :** $f(x)=\begin{cases} \log_e[x], & 1\le x<3 \\ |\log_e x|, & 3\le x<4 \end{cases}$

The function can be re-written as

$$f(x)=\begin{cases} 0, & \text{when } 1\le x<2 \\ \log_e 2, & \text{when } 2\le x<3 \\ \log_e x, & \text{when } 3\le x<4 \end{cases}$$

$\therefore$ $f(x)$ is continuous and differentiable everywhere except at 2, 3.

$f(2 - 0) = 0, f(2 + 0) = \log_e 2$
$\Rightarrow$ $f(x)$ is not continuous at $x = 2$
$f(3 - 0) = \log_e 2, f(3 + 0) = \log_e 3$
$\Rightarrow$ $f(x)$ is not continuous at $x = 3$
$\therefore$ $f(x)$ is not continuous at $x = 2, 3$, *i.e.* the graph is broken at $x = 2, 3$
$\Rightarrow$ $f(x)$ is not differentiable at $x = 2, 3$, *i.e.* the graph does not have a definite tangent at $x = 2, 3$.

**77. (b, c) :** The probability of $A$ winning

$= P(A) + P(A^C \cap B^C \cap A) + P(A^C \cap B^C \cap A^C \cap B^C \cap A) + \ldots$

$$=\frac{1}{2}+\left(1-\frac{1}{2}\right)\left(1-\frac{1}{2}\right)\frac{1}{2}+\ldots.$$

$$=\frac{1}{2}+\left(\frac{1}{2}\right)^3+\ldots=\frac{1/2}{1-\frac{1}{4}}=\frac{2}{3}$$

$\therefore$ Probability of $B$ winning $= 1-\frac{2}{3}=\frac{1}{3}$

$\therefore$ The expectation of $A$ $=$ ₹ $30\times\frac{2}{3}=$ ₹ $20$

and the expectation of $B$ $=$ ₹ $30\times\frac{1}{3}=$ ₹ $10$

**78. (a, c, d) :** Here $f(x)=x^2+\frac{\lambda}{x}$

$\therefore$ $f'(x)=2x-\frac{\lambda}{x^2}$ and $f''(x)=2+\frac{2\lambda}{x^3}$

Now, $f'(x) = 0$ gives $x=\left(\frac{\lambda}{2}\right)^{1/3}$

If $\lambda = 16$ then $x = 2$ and $f''(x)|_{x=2} = 2 + 4 = 6 > 0$.
Hence, $f(x)$ has a minimum value at $x = 2$.
Thus, $f(x)$ has a maximum value for no real value of $\lambda$.
When $\lambda = -1$, $f''(x) = 0$ if $x = 1$, so $f(x)$ has a point of inflection at $x = 1$.

**79. (a, c) :** $I_n=\int_0^{\pi/4}\tan^n x\,dx$

$$=\int_0^{\pi/4}\tan^{n-2}x(\sec^2 x-1)dx$$

$$=\int_0^{\pi/4}\tan^{n-2}x\sec^2 x\,dx-\int_0^{\pi/4}\tan^{n-2}x dx$$

$$=\int_0^{\pi/4}\tan^{n-2}x d(\tan x)-\int_0^{\pi/4}\tan^{n-2}x dx$$

$$=\left.\frac{\tan^{n-1}x}{n-1}\right|_0^{\pi/4}-I_{n-2}=\frac{1}{n-1}-I_{n-2}$$

$$\therefore\ I_n=\frac{1}{n-1}-I_{n-2}$$

$\therefore$ $I_5=\frac{1}{4}-I_3$ and $I_3=\frac{1}{2}-I_1 \Rightarrow I_1 = I_3 + 2I_5$

**80. (a, b) :** Let $g'(1) = a, g''(2) = b$.
Then $f(x) = x^2 + ax + b$
$\therefore$ $g(x) = (1 + a + b)x^2 + (2x + a)x + 2$
$= (3 + a + b)x^2 + ax + 2$
$\therefore$ $g'(x) = 2(3 + a + b)x + a$
Hence $a = 2(3 + a + b)\cdot 1 + a$ *i.e.* $3 + a + b = 0$
and $b = 2(3 + a + b)$ *i.e.* $b + 2a + 6 = 0$
Hence, $b = 0$, $a = -3$.
So $f(x) = x^2 - 3x$ and $g(x) = -3x + 2$

# PRACTICE PAPER 2016

# JEE ADVANCED

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## SINGLE ANSWER CORRECT TYPE

**1.** In the triangle $ABC$ the medians from $B$ and $C$ are perpendicular. The value of $\cot B + \cot C$ cannot be

(a) $\frac{7}{3}$ (b) $\frac{5}{3}$

(c) $\frac{4}{3}$ (d) none of these

**2.** The position vectors of the vertices $A$, $B$, $C$ of a tetrahedron $ABCD$ are $\hat{i}+\hat{j}+\hat{k}, \hat{i}$ and $3\hat{i}$ respectively and the altitude from the vertex $D$ to the opposite face $ABC$ meets the face at $E$. If the length of the edge $AD$ is 4 and the volume of the tetrahedron is $\frac{2\sqrt{2}}{3}$, then the length of $DE$ is

(a) 1 (b) 2 (c) 3 (d) 4

**3.** Number of solutions of the equation $4\sin^2 x + \tan^2 x + \cot^2 x + \operatorname{cosec}^2 x = 6$ in $[0, \pi]$ is

(a) 0 (b) 2 (c) 8 (d) 4

**4.** The area of the loop of the curve $y^2 = x^4 (x + 2)$ is [in square units]

(a) $\frac{32\sqrt{2}}{105}$ (b) $\frac{64\sqrt{2}}{105}$

(c) $\frac{128\sqrt{2}}{105}$ (d) $\frac{256\sqrt{2}}{105}$

**5.** The number of integer values of $p$ for which the vectors $\hat{i}+p^2\hat{j}+p^2\hat{k}$, $p^2\hat{i}+\hat{j}+p^4\hat{k}$ and $p^4\hat{i}+p^4\hat{j}+\hat{k}$ are coplanar is

(a) 8 (b) 4

(c) 1 (d) none of these

**6.** Area bounded by the curves $y = e^x$, $y = \log_e x$ and the lines $x = 0$, $y = 0$, $y = 1$ is

(a) $e^2 + 2$ (b) $e^2 + 1$

(c) $e + 2$ (d) $e - 1$

**7.** Three positive real numbers $x, y, z$ satisfy the equations $x^2+\sqrt{3}\,xy+y^2=25$; $y^2+z^2=9$ and $x^2 + xz + z^2 = 16$. Then the value of $xy+2yz+\sqrt{3}\,xz$ is

(a) 18 (b) 24 (c) 30 (d) 36

**8.** The solution of the differential equation $2x^3ydy + (1 - y^2)(x^2y^2 + y^2 - 1)dx = 0$ [Where $c$ is a constant]

(a) $x^2y^2 = (cx + 1)(1 - y^2)$

(b) $x^2y^2 = (cx + 1)(1 + y^2)$

(c) $x^2y^2 = (cx - 1)(1 - y^2)$

(d) none of these

**9.** Area of the region (in sq. units) in which point $P(x, y)$, $\{x > 0\}$ lies; such that $y \le \sqrt{16-x^2}$ and $\left|\tan^{-1}\left(\frac{y}{x}\right)\right| \le \frac{\pi}{3}$ is

(a) $\frac{16}{3}\pi$ (b) $\frac{8\pi}{3}+8\sqrt{3}$

(c) $4\sqrt{3}-\pi$ (d) $\sqrt{3}-\pi$

**10.** A circle of radius 4 cm is inscribed in $\Delta ABC$, which touches the side $BC$ at $D$. If $BD = 6$ cm, $DC = 8$ cm then which is false ?

(a) the triangle is necessarily obtuse angled triangle

(b) $\tan\frac{A}{2}=\frac{4}{7}$

(c) perimeter of the triangle $ABC$ is 42 cm

(d) area of $\Delta ABC$ is 84 cm2

**11.** The reflection of the hyperbola $xy = 1$ in the line $y = 2x$ is the curve $12x^2 + rxy + sy^2 + t = 0$ then the value of '$r$' is

(a) – 7 (b) 25 (c) –175 (d) 90

**12.** An ellipse with major and minor axes length $10\sqrt{3}$ and 10 respectively slides along the co-ordinate axes

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and always remains confined in the first quadrant. The length of the arc of the locus of the centre of the ellipse is

(a) 10p (b) 5p (c) $\frac{5\pi}{4}$ (d) $\frac{5\pi}{3}$

**13.** The equation of common tangent at the point of contact of two parabolas $y^2 = x$ and $2y = 2x^2 - 5x + 1$ is

(a) $x + y + 1 = 0$ (b) $x + 2y + 1 = 0$

(c) $x - 2y - 1 = 0$ (d) $-x + 2y - 1 = 0$

**14.** The range of '$a$', for which a circle will pass through the points of intersection of the hyperbola $x^2 - y^2 = a^2$ and the parabola $y = x^2$, is

(a) $a \in (-3, -2)$ (b) $a \in (-1, 1)$

(c) $a \in (2, 4)$ (d) $a \in (4, 6)$

**15.** Two tangents are drawn to an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ from a point $P(h, k)$; if the points at which these tangents meet the axes of the ellipse be concyclic, then the locus of $P$ is

(a) an ellipse (b) rectangular hyperbola

(c) parabola (d) circle

**16.** The inclination to the major axis of the diameter of an ellipse the square of whose length is the harmonic mean between the squares of the major and minor axes is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{2}$

**17.** The number of the functions $f$ from the set $X = \{1, 2, 3\}$ to the $Y = \{1, 2, 3, 4, 5, 6, 7\}$ such that $f(i) \le f(j)$ for $i < j$ and $i, j \in X$ is

(a) ${}^6C_3$ (b) ${}^7C_3$

(c) ${}^8C_3$ (d) ${}^9C_3$

**18.** Eight straight lines are drawn in the plane such that no two lines are parallel and no three lines are concurrent. The number of parts into which these lines divide the plane is

(a) 29 (b) 32 (c) 36 (d) 37

**19.** How many combinations can be made up of 3 hens, 4 ducks and 2 geese so that each combination has hens, ducks and geese? (birds of same kind all different)

(a) 305 (b) 315 (c) 320 (d) 325

**20.** Let '$C$' denote the set of complex numbers and define $A$ & $B$ by $A = \{(z, w); z, w, \in C$ and $|z| = |w|\}$
$B = \{(z, w); z, w, \in C$ and $z^2 = w^2\}$ then

(a) $A = B$ (b) $A \subset B$

(c) $B \subset A$ (d) none of these

**21.** If $f(x) = \prod_{i=1}^{i=3}(x-a_i)+\sum_{i=1}^{3}a_i - 3x$, where $a_i < a_{i+1}$, then $f(x) = 0$ has

(a) only one real root

(b) three real roots of which two of them are equal

(c) three distinct real roots

(d) three equal roots

## MULTIPLE ANSWER CORRECT TYPE

**22.** The projection of line $3x - y + 2z - 1 = 0 = x + 2y - z - 2$ on the plane $3x + 2y + z = 0$ is

(a) $\frac{x+1}{11}=\frac{y-1}{-9}=\frac{z-1}{-15}$

(b) $3x - 8y + 7z + 4 = 0 = 3x + 2y + z$

(c) $\frac{x+12}{11}=\frac{y+8}{-9}=\frac{z+14}{15}$

(d) $\frac{x+12}{11}=\frac{y+8}{-9}=\frac{z+14}{-15}$

**23.** Which of the following functions will not have absolute minimum value?

(a) cot (sin$x$) (b) tan (log $x$)

(c) $x^{2005} - x^{1947} + 1$ (d) $x^{2006} + x^{1947} + 1$

**24.** $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ intersects the co-ordinate axes at points $A$, $B$ and $C$ respectively. If $\Delta PQR$ has mid-points $A$, $B$ and $C$ then

(a) centroids of $\Delta ABC$ and $\Delta PQR$ coincide

(b) foot of normal to $\Delta ABC$ from $O$ is circumcentre of $\Delta PQR$

(c) $ar(\Delta PQR)=2\sqrt{a^2b^2+b^2c^2+c^2a^2}$

(d) incentres of $\Delta ABC$ and $\Delta PQR$ coincide

**25.** If in a triangle $ABC$, cos$A$ cos$B$ + sin$A$ sin$B$ = Sin$^3C$ = 1, then, with usual notation in $\Delta ABC$,

(a) the triangle is isosceles

(b) the triangle is right angled

(c) $R:r=(\sqrt{2}+1):1$

(d) $r_1:r_2:r_3=1:1:(\sqrt{2}+1)$

**26.** If the orthocenter of an isosceles triangle lies on the incircle of the triangle then

(a) the base angle of the triangle is $\cos^{-1}\frac{2}{3}$

(b) the triangle is acute

(c) the base angle of the triangle is $\tan^{-1}\frac{\sqrt{5}}{2}$

(d) If $S$, $I$ are the circumcentre and incentre and $R$ is circumradius then $\frac{SI}{R}=\frac{1}{3}$

**27.** A circle touches the parabola $y^2=2x$ at $P\left(\frac{1}{2},1\right)$ and cuts the parabola at its vertex V. If the centre of the circle is Q, then

(a) The radius of the circle is $5/\sqrt{2}$

(b) The radius of the circle is the maximum value of $\frac{7}{2}\sin 3x+\frac{1}{2}\cos 3x$

(c) Area of $\Delta PVQ$ is $\frac{15}{16}$

(d) Slope of $PQ$ is $-2$

**28.** Thirteen persons are sitting in a row. Number of ways in which four persons can be selected so that no two of them are consecutive is equal to

(a) number of ways in which all the letters of the word "M A R R I A G E" are permutated if no two vowels are never together.

(b) number of numbers lying between 100 and 1000 using only the digits 1, 2, 3, 4, 5, 6, 7 without repetition.

(c) number of ways in which 4 alike chocolates can be distributed among 10 children so that each child getting at most one chocolate.

(d) number of triangles can be formed by joining 12 points in a plane, of which 5 are collinear.

**29.** If $a_1, a_2, a_3, \ldots a_n$ is sequence of positive numbers which are in A.P. with common difference '$d$' and $a_1+a_4+a_7+\ldots\ldots+a_{16}=147$ then

(a) $a_1+a_6+a_{11}+a_{16}=98$

(b) $a_1+a_{16}=49$

(c) $a_1+a_4+a_7+\ldots\ldots a_{16}=6a_1+45d$

(d) Maximum value of $a_1a_2\ldots\ldots a_{16}$ is $\left(\frac{49}{2}\right)^{16}$

**30.** Suppose three real numbers $a$, $b$, $c$ are in G.P. Let $z=\frac{a+ib}{c-ib}$ then

(a) $z=\frac{ib}{c}$ (b) $z=\frac{ia}{b}$

(c) $z=\frac{ia}{c}$ (d) $z=0$

**31.** The number of ways in which we can choose 2 distinct integers from 1 to 200 so that the difference between them is atmost 20 is

(a) 3790 (b) ${}^{200}C_2-{}^{180}C_2$

(c) ${}^{180}C_1\times 20+\frac{19\times 20}{2}$ (d) ${}^{180}C_2$

**32.** In a gambling between Mr. A and Mr. B a machine continues tossing a fair coin until the two consecutive throws either HT or TT are obtained for the first time. If it is HT, Mr. A wins and if it is TT, Mr. B wins. Which of the following is (are) true?

(a) probability of winning Mr.A is $\frac{3}{4}$

(b) Probability of Mr.B winning is $\frac{1}{4}$

(c) Given first toss is head probability of Mr. A winning is 1

(d) Given first toss is tail, probability of Mr.A winning is $\frac{1}{2}$

## COMPREHENSION TYPE

### Paragraph for Question No. 33 to 35

Let the curves $S_1: y=x^2$, $S_2: y=-x^2$, $S_3: y^2=4x-3$

**33.** Area bounded by the curves $S_1$, $S_2$, $S_3$ is

(a) $\frac{4}{3}$ sq. units (b) $\frac{8}{3}$ sq. units

(c) $\frac{1}{6}$ sq. units (d) $\frac{1}{3}$ sq. units

**34.** Area bounded by the curves $S_1$, $S_3$ and the line $x=3$ is

(a) $\frac{13}{3}$ sq. units (b) $\frac{5}{4}$ sq. units

(c) $\frac{8}{3}$ sq. units (d) $\frac{7}{4}$ sq. units

**35.** Area bounded by the curve $S_3$, $y\le -1$ and the line $x=3$ is

(a) $\frac{7}{3}$ sq. units (b) $\frac{11}{3}$ sq. units

(c) $\frac{9}{2}$ sq. units (d) $\frac{13}{4}$ sq. units

### Paragraph for Question No. 36 to 38

Let $PQRS$ be a rectangle of size 9 × 3, if it is folded along $QS$ such that plane $PQS$ is perpendicular to plane $QRS$ and point $R$ moves to point $T$.

**36.** Distance between the points $P$ and $T$ will be

(a) $\sqrt{90}$ (b) $\frac{3}{5}\sqrt{205}$

(c) $\frac{4}{5}$ (d) none of these

**37.** If $\theta$ is angle between the line $QP$ and $QT$ then $\tan\theta$ is equal to

(a) $\frac{3}{10}$ (b) $\frac{10}{3}$

(c) $\frac{\sqrt{91}}{3}$ (d) none of these

**38.** Shortest distance between the edges $PQ$ and $TS$ is

(a) $3\sqrt{\frac{10}{19}}$ (b) $\sqrt{\frac{10}{19}}$

(c) $2\sqrt{\frac{10}{19}}$ (d) none of these

**Paragraph for Question No. 39 to 41**

At times the methods of coordinates becomes effective in solving problems of properties of triangles. We may choose one vertex of the triangle as origin and one side passing through this vertex as $x$-axis. Thus without loss of generality, we can assume that every triangle $ABC$ has a vertex situated at (0,0) another at $(x,0)$ and third one at $(h,k)$.

**39.** If in $\Delta ABC$, $AC = 3$, $BC = 4$ medians $AD$ and $BE$ are perpendicular then area of $\Delta ABC$ (in sq. units) is

(a) $\sqrt{7}$ (b) $\sqrt{11}$ (c) $2\sqrt{2}$ (d) $2\sqrt{11}$

**40.** Suppose the bisector $AD$ of the interior angle $A$ of $\Delta ABC$ divides side $BC$ into segments $BD = 4$; $DC = 2$ then

(a) $b > c$ and $c < 4$

(b) $2 < b < 6$ and $c < 1$

(c) $2 < b < 6$ and $4 < c < 12$

(d) $b < c$ and $c > 4$

**41.** If in the above question, altitude $AE > \sqrt{10}$ and suppose lengths of $AB$ and $AC$ are integers, then $b$ will be

(a) 3 (b) 6

(c) 4 or 5 (d) 3 or 6

**Paragraph for Question No. 42 to 44**

In a $\Delta ABC$ $B(2, 4)$ , $C(6, 4)$ and $A$ lies on a curve $S$ such that $\tan\frac{B}{2}\tan\frac{C}{2} = \frac{1}{2}$

**42.** Let a line passing through $C$ and perpendicular to $BC$ intersects the curve $S$ at $P$ and $Q$. If $R$ is the mid point of $BC$ then area of $\Delta PQR$ is

(a) $\frac{18}{3}$ sq. units (b) $\frac{8}{3}$ sq. units

(c) $\frac{32}{3}$ sq. units (d) $\frac{26}{3}$ sq. units

**43.** From the data of the above problem the radius of the circle passing through $P$, $B$, $C$ is

(a) $\frac{5}{3}$ units (b) $\frac{9}{4}$ units

(c) $\frac{16}{3}$ units (d) none of these

**44.** The eccentricity of the hyperbola whose transverse axis lies along the line through $B$, $C$ and the parabola passes through $B$, $C$ and (0, 2) is

(a) $\frac{\sqrt{19}}{4}$ (b) $\frac{\sqrt{17}}{2}$

(c) $\sqrt{\frac{7}{3}}$ (d) $\frac{2}{\sqrt{3}}$

## MATRIX MATCH TYPE

**45.** Match the following

| | Column I | | Column II |
|---|---|---|---|
| (A) | Number of solutions of $\sin x = \frac{x}{10}$ is | (p) | 1 |
| (B) | Number of ordered pairs $(x, y)$ satisfying $\lvert x\rvert+\lvert y\rvert=2, \sin\left(\frac{\pi x^2}{3}\right)=1$ is | (q) | 4 |
| (C) | Number of solution of the equation $\sin\left(\frac{\pi x}{2\sqrt{3}}\right)=x^2-2\sqrt{3}x+4$ | (r) | 7 |
| (D) | The number of ordered pairs $(x, y)$ satisfying the equation $\sin x + \sin y = \sin(x + y)$ and $\lvert x\rvert + \lvert y\rvert = 1$ is | (s) | 6 |

**46.**

| | Column I | | Column II |
|---|---|---|---|
| (A) | If slope of tangents from $(-1, 2)$ to parabola $y^2 = 8x$ are $m_1$ and $m_2$ then $\frac{1}{m_1}+\frac{1}{m_2}-1$ is | (p) | 2 |
| (B) | Let $S_1$, $S_2$ are foci of an ellipse whose eccentricity is $e$. If $P$ is extremity of minor axis such that $\angle S_1PS_2 = 90°$, then $4e^2$ is | (q) | 4 |
| (C) | If foot of perpendicular from focus upon any tangent of parabola $y^2 - 4y + 4x + 8 = 0$ lies on the line $x + k = 0$ then $k$ is | (r) | 0 |

| | | | |
|---|---|---|---|
| (D) | Let hyperbola has eccentricity 2 and its conjugate hyperbola having $e$ then $3e^2$ is | (s) | 1 |

**47.** We are given $M$ urns, numbered 1 to $M$ and $n$ balls ($n < M$) and $P(A)$ denote the probability that each of the urns numbered 1 to $n$, will contain exactly one ball.

| Column I | | Column II | |
|---|---|---|---|
| (A) | If the balls are different and any number of balls can go to any urn then $P(A)$= | (p) | $\frac{1}{^{M}C_n}$ |
| (B) | If the balls are identical and any number of balls can go to any urn then $P(A)$= | (q) | $\frac{1}{^{(M+n-1)}C_{M-1}}$ |
| (C) | If the balls are identical but at most one ball can be put in any box, then $P(A)$= | (r) | $\frac{n!}{^{M}C_n}$ |
| (D) | If the balls are different and at most one ball can be put in any box, then $P(A)$= | (s) | $\frac{n!}{M^n}$ |

**48.** '$n$' whole numbers are randomly chosen and multiplied, then probability that

| Column I | | Column II | |
|---|---|---|---|
| (A) | the last digit is 1, 3, 7 or 9 | (p) | $\frac{8^n-4^n}{10^n}$ |
| (B) | the last digit is 2, 4, 6, 8 | (q) | $\frac{5^n-4^n}{10^n}$ |
| (C) | the last digit is 5 | (r) | $\frac{4^n}{10^n}$ |
| (D) | the last digit is zero | (s) | $\frac{10^n-8^n-5^n+4^n}{10^n}$ |

## INTEGER TYPE

**49.** If the area bounded by the curves $y = -x^2 + 6x - 5$, $y = -x^2 + 4x - 3$ and the line $y = 3x - 15$ is $\frac{73}{\lambda}$, then the value of $\lambda$ is

**50.** With usual notation in triangle $ABC$, the numerical value of $\left(\frac{a+b+c}{r_1+r_2+r_3}\right)\left(\frac{a}{r_1}+\frac{b}{r_2}+\frac{c}{r_3}\right)$ is

**51.** The number of solutions of the equation $\sin^{-1}\frac{1+x^2}{2x}=\frac{\pi}{2}\sec(x-1)$ are

**52.** Let $R = \{x, y : x^2 + y^2 \le 144 \text{ and } \sin(x + y) \ge 0\}$. And $S$ be the area of region given by $R$, then find $S/9\pi$.

**53.** Number of real values of $x$, satisfying the equation $[x]^2 - 5[x] + 6 - \sin x = 0$ ($[\cdot]$ denoting the greatest integer function) is

**54.** The point $P(1,2,3)$ is reflected in the $xy$-plane, then its image $Q$ is rotated by 180° about the $x$-axis to produce $R$, and finally $R$ is translated in the direction of the positive $y$-axis through a distance $d$ to produce $S$ (1, 3, 3). The value of $d$ is

**55.** Shortest distance between the $z$-axis and the line $x + y + 2z - 3 = 0 = 2x + 3y + 4z - 4$ is ____

**56.** The minimum distance of $4x^2 + y^2 + 4x - 4y + 5 = 0$ from the line $-4x + 3y = 3$ is ____________

**57.** The tangents drawn from the origin to the circle $x^2 + y^2 - 2rx - 2hy + h^2 = 0$ are perpendicular then sum of all possible values of $\frac{h}{r}$ is ____________

**58.** Two lines $zi-\bar{z}i+2=0$ and $z(1+i)+\bar{z}(1-i)+2=0$ intersect at a point $P$. There is a complex number $\alpha = x + iy$ at a distance of 2 units from the point $P$ which lies on line $z(1+i)+\bar{z}(1-i)+2=0$. Find $[|x|]$ (where $[\cdot]$ represents greatest integer function).

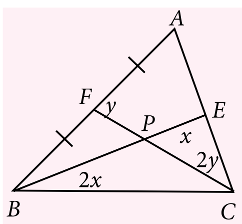
**59.** Let $x$ be in radians with $0<x<\frac{\pi}{2}$. If $\sin(2\sin x) = \cos(2\cos x)$; then $\tan x + \cot x$ can be written as $\frac{a}{\pi^c-b}$ where $a, b, c \in N$. Then the value of $\left(\frac{a+b+c}{25}\right)$ is

## SOLUTIONS

1. **(d) :** $\tan B = \dfrac{\dfrac{y}{2x}+\dfrac{y}{x}}{1-\dfrac{y^2}{2x^2}} = \dfrac{3xy}{2x^2-y^2}$

$\cot B = \dfrac{2x^2-y^2}{3xy}, \cot C = \dfrac{2y^2-x^2}{3xy}$

$\cot B + \cot C = \dfrac{x^2+y^2}{3xy} \geq \dfrac{2}{3}$




$\tan \angle PBF = \dfrac{y}{2x}$

$\tan \angle PBC = \dfrac{y}{x}$

2. **(b) :** $\overleftrightarrow{BC}$ is the $x$-axis and $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$

Area of $\Delta ABC = \dfrac{1}{2}\times\sqrt{2}\times 2 = \sqrt{2}$

Volume $= \dfrac{1}{3}\times\sqrt{2}\times DE = \dfrac{2\sqrt{2}}{3} \Rightarrow DE = 2$

3. **(b) :** $\tan x = t \Rightarrow \dfrac{4t^2}{1+t^2}+t^2+\dfrac{2}{t^2}=5$

$\Rightarrow \ (t^2-1)\,(t^4+t^2-2)=0$

$\Rightarrow \ t^2 = 1$

$\Rightarrow \ x = \dfrac{\pi}{4}, \dfrac{3\pi}{4}$

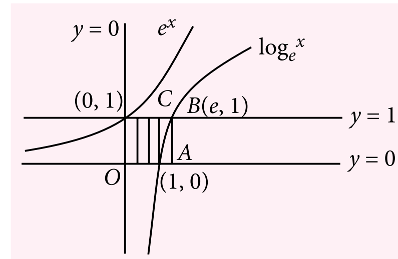
4. **(d) :** Area $= 2\int\limits_{-2}^{0} y\,dx = 2\int\limits_{-2}^{0} x^2\sqrt{x+2}\,dx$

$= 4\int\limits_{0}^{\sqrt{2}} (z^2-2)^2 z^2\,dz$ (where $\sqrt{x+2}=z$)

$= 4\left[\dfrac{z^7}{7}-\dfrac{4z^5}{5}+\dfrac{4z^3}{3}\right]_0^{\sqrt{2}} = \dfrac{256\sqrt{2}}{105}$

5. **(d) :** $\begin{vmatrix} 1 & p^2 & p^2 \\ p^2 & 1 & p^4 \\ p^4 & p^4 & 1 \end{vmatrix} = 0$

6. **(d) :**



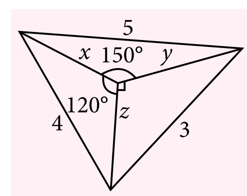

Area = area of rectangle $OABC - \int\limits_1^e \log_e x\,dx$

$= e-(x\ln x - x)\Big|_1^e = e-1$

7. **(b) :** Given

$x^2+y^2-2xy\cos\dfrac{5\pi}{6}=25; x^2+z^2-2xz\cos\dfrac{2\pi}{3}=16$

and $y^2 + z^2 = 9$.




Area $= \dfrac{1}{2}xy\cdot\dfrac{1}{2}+yz\cdot\dfrac{1}{2}+\dfrac{1}{2}xz\cdot\dfrac{\sqrt{3}}{2}=6$

$\therefore \ xy + 2yz + \sqrt{3}\,zx = 24$

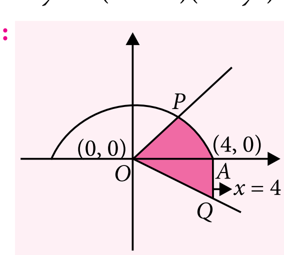
8. **(c) :** $\dfrac{2y}{(1-y^2)^2}\cdot\dfrac{dy}{dx}+\dfrac{y^2}{(1-y^2)}\cdot\dfrac{1}{x}=\dfrac{1}{x^3}$

Put $\dfrac{y^2}{1-y^2}=t \ \Rightarrow \ \dfrac{2y}{(1-y^2)^2}\,\dfrac{dy}{dx}=\dfrac{dt}{dx}$

$\Rightarrow \ \dfrac{dt}{dx}+\dfrac{t}{x}=\dfrac{1}{x^3} \quad \Rightarrow \ t\cdot x = \int\dfrac{1}{x^2}dx + c$

$\Rightarrow \ x^2y^2 = (cx-1)(1-y^2)$

9. **(b) :**




$\left|\tan^{-1}\dfrac{y}{x}\right| \leq \dfrac{\pi}{3}$ is equivalent to $-\sqrt{3} \leq \dfrac{y}{x} \leq \sqrt{3}$

Required area is the area of shaded region ($APOQ$)

= area of $\Delta OAQ$ + area of sector ($OAP$)

$= \dfrac{1}{2}\times 4\times 4\sqrt{3}+\dfrac{\pi(4\times 4)}{6} \quad = \left(\dfrac{8\pi}{3}+8\sqrt{3}\right)$

**10. (a) :** $\tan\frac{B}{2}=\frac{2}{3}, \tan\frac{C}{2}=\frac{1}{2} \Rightarrow \tan\frac{A}{2}=\frac{4}{7}$

$\tan\frac{B}{2}.\tan\frac{C}{2}=\frac{s-a}{s} \Rightarrow 2s=3a=42$

$\therefore$ Perimeter = 42 cm

$\therefore$ $\Delta = r.s. = 84$ cm$^2$

$\therefore$ $\tan\frac{A}{2}, \tan\frac{B}{2}, \tan\frac{C}{2}$ all are less than 1. All angles are acute.

**11. (b) :** The reflection of $\alpha$, $\beta$ in the line $ax + by + c = 0$ is given by $\frac{x-\alpha}{a}=\frac{y-\beta}{b}=\frac{-2(a\alpha+b\beta+c)}{a^2+b^2}$.

The reflection of $(\alpha, \beta)$ in the line $y = 2x$ is

$(\alpha_1, \beta_1)=\left(\frac{4\beta-3\alpha}{5}, \frac{4\alpha+3\beta}{5}\right)$ Also, $\alpha_1\beta_1=1$

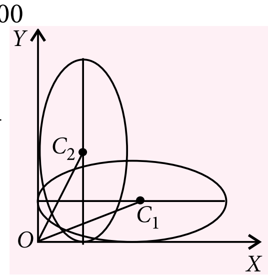
$\Rightarrow$ $12\alpha^2 - 7\alpha\beta - 12\beta^2 + 25 = 0$

**12. (d) :** The locus of the centre of the ellipse is director circle *i.e.* $x^2 + y^2 = 100$

$\angle C_1OC_2 = \theta$

$\Rightarrow \frac{\pi}{2}-2\tan^{-1}\left(\frac{5}{5\sqrt{3}}\right)=\frac{\pi}{6}$

$\therefore$ arc length $=10.\frac{\pi}{6}=\frac{5\pi}{3}$




**13. (b) :** $y^2 = x$ ...(i)

$2y = 2x^2 - 5x + 1$ ..(ii)

Solving (i) & (ii)

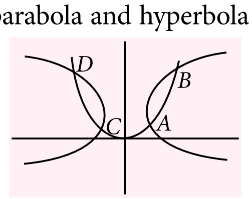
$(y + 1)^2 (2y^2 - 4y + 1) = 0$

$\therefore$ $y = -1$ is the repeated root which is the $y$ co-ordinate of point of contact. So point of contact = $(1, -1)$.

$\therefore$ Common tangent at point of contact

$y(-1)=\frac{x+1}{2}$ or $x + 2y + 1 = 0$.

**14. (b) :** Family of curves passing through the intersection of the parabola and hyperbola is




$x^2 - y^2 - a^2 + \lambda (x^2 - y) = 0$.

*i.e.*$(1 + \lambda)x^2 - y^2 - \lambda y - a^2 = 0$

For this equation to represent a circle

$1 + \lambda = -1 \Rightarrow \lambda = -2$

$\therefore$ $x^2 + y^2 - 2y + a^2 = 0$

Moreover this equation represents a real circle, if $g^2 + f^2 - c > 0$.

$\Rightarrow$ $0 + 1 - a^2 > 0 \Rightarrow a \in (-1, 1)$

**15. (b) :** The equation of pair of tangents from $P(h, k)$ to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

$$\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1\right)\left(\frac{h^2}{a^2}+\frac{k^2}{b^2}-1\right)=\left(\frac{xh}{a^2}+\frac{yk}{b^2}-1\right)^2 \quad ...(i)$$

Let these tangents meet the $x$-axis at the points $A_1$, $A_2$ putting $y = 0$ in (i), we have

$$\frac{x^2}{a^2}\left(\frac{k^2}{b^2}-1\right)+2\frac{xh}{a^2}-\left(\frac{h^2}{a^2}+\frac{k^2}{b^2}\right)=0 \quad ...(ii)$$

$OA_1$ & $OA_2$ are the roots of (ii)

$$\therefore \quad OA_1 \,.\, OA_2 = \frac{\left(\frac{h^2}{a^2}+\frac{k^2}{b^2}\right)a^2}{1-\frac{k^2}{b^2}} \quad ...(iii)$$

If these tangents meet the $y$-axis at $B_1$, $B_2$ then putting $x = 0$ in (i), we get

$$OB_1 \,.\, OB_2 = \frac{\left(\frac{h^2}{a^2}+\frac{k^2}{b^2}\right)b^2}{1-\frac{h^2}{a^2}}$$

$\because$ $OA_1$, $OA_2$, $OB_1$, $OB_2$ are concyclic.

$\therefore$ $OA_1 \cdot OA_2 = OB_1 \cdot OB_2$

$$\left(1-\frac{h^2}{a^2}\right)a^2=b^2\left(1-\frac{k^2}{b^2}\right)$$

or $x^2 - y^2 = a^2 - b^2$.

**16. (a) :** $4(a^2\cos^2\theta+b^2\sin^2\theta)=\frac{2(4a^2)(4b^2)}{4a^2+4b^2}$

(Standard formula)

**17. (d) :** ${}^7C_3 + 2 \times {}^7C_2 + {}^7C_1 = {}^9C_3$.

**18. (d) :** Find $t_n$ in 2, 4, 7, 11.......

$t_n=1+\frac{n(n+1)}{2}$

**19. (b) :** $(2^3 - 1)(2^4 - 1)(2^2 - 1)$

**20 (c) :** $z^2 = w^2$ taking modulus on both sides $|z|=|w|$

Thus, $B \subset A$

**21. (c) :** $f(x) = (x - a_1)(x - a_2)(x - a_3) + (a_1 - x) + (a_2 - x) + (a_3 - x)$

Now $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$.

Again $f(a_1) = (a_2 - a_1) + (a_3 - a_1) > 0$

$[\because a_1 < a_2 < a_3]$

$\Rightarrow$ One root belongs to $(-\infty, a_1)$

Also, $f(a_3) = (a_1 - a_3) + (a_2 - a_3) < 0$

$\Rightarrow$ One root belongs to $(a_1, a_3)$

So $f(x) = 0$ has three distinct real roots.

**22. (a) :** Equation of a plane passing through the line $3x - y + 2z - 1 = 0 = x + 2y - z - 2$ is $3x - y + 2z - 1 + \lambda(x + 2y - z - 2) = 0$

Since it is perpendicular to the given plane

$\therefore\ \lambda = -\frac{3}{2}$

(Take the dot product of d.c. of both normal)

Equation of the line of projection is

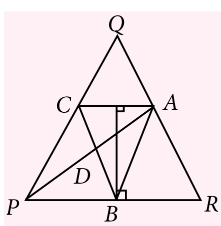
$3x - 8y + 7z + 4 = 0 = 3x + 2y + z$

Its direction ratios are $<11, -9, -15>$ and the point $(-1, 1, 1)$ lies on the line

$\therefore\ \frac{x+1}{11} = \frac{y-1}{-9} = \frac{z-1}{-15}$ is also the equation of the line of projection.

**23. (d) :** Even degree polynomial with leading coefficient +ve will have absolute minimum.

**24. (a, b, c) :**




$AC || PR$ and $2AC = PR$

So, $ABPC$ is a parallelogram comparing the coordinates of mid-point of diagonals, we get $P(-a, b, c)$ and $Q(a, -b, c)$ and $R(a, b, -c)$

Also, $AD$ and $AP$ are median of $\Delta ABC$ and $\Delta PQR$ respectively. So, centroids are coinciding. The perpendicular bisector of $PR$ is also perpendicular to $AC$. Therefore circumcentre of $\Delta PQR$ is orthocenter of $\Delta ABC$.

$ar\Delta PQR = 4\ ar\Delta ABC$

$= 4\sqrt{(OAB)^2 + (OBC)^2 + (OAC)^2}$

where $OAB$ is the area of the projection of $\Delta ABC$ on the plane $XOZ$ etc.

**25. (a, b, c, d) :** The given relation implies cos (A – B) =1 and so, A = B and C = 90°

**26. (a) :** Let $ABC$ be the triangle in which $AB = AC$. Let $I$, $P$ respectively be the incentre and the orthocenter of the triangle.

(b) $AI = r\operatorname{cosec}\frac{A}{2}, AP = 2R\cos A$

(c) $r\operatorname{cosec}\frac{A}{2} = 2R\cos A + r$

**27. (a, b, c, d) :** The circle is the tangent to the parabola at $\left(\frac{1}{2}, 1\right)$ and the equation of tangent is $2x - 2y + 1 = 0$

we can write the family of circle in the form

$\left(x - \frac{1}{2}\right)^2 + (y-1)^2 + \lambda(2x - 2y + 1) = 0$

**28. (b, c, d) :** $x_1 + x_2 + x_3 + x_4 + x_5 = 9$, $x_1, x_5 \ge 0$ $x_2, x_3, x_4 \ge 1$, number of solutions are 210.

(a) $5 \times 12 \times 12 = 720$ (b) ${}^7P_3 = 210$

(c) ${}^{10}C_4 = 210$ (d) ${}^{12}C_3 - {}^5C_3 = 210$

**29. (a, b, c, d) :** $a_1 + a_4 + a_7 + .... a_{16} = 147$

$\Rightarrow\ 3(a_1 + a_{16}) = 147 \Rightarrow a_1 + a_{16} = 49$

Again $a_1 + a_4 + a_7 + a_{10} + ...... + a_{16}$

$= a_1 + a_1 + 3d + a_1 + 6d + ...... + a_1 + 15d$

$= 6a_1 + 45d = 147$

$\Rightarrow\ 2a_1 + 15d = 49$

$a_1 + a_6 + a_{11} + a_{16} = a_1 + a_1 + 5d + a_1 + 10d + a_1 + 15d$

$= 4a_1 + 30d$

$= 2(2a_1 + 15d) = 2(49) = 98$

Now using AM $\ge$ GM

$$\frac{a_1 + a_2 + ... + a_{16}}{16} \ge (a_1 a_2 a_3 ... a_{16})^{\frac{1}{16}}$$

$$\frac{8(a_1 + a_{16})}{16} \ge (a_1 a_2 a_3 ... a_{16})^{\frac{1}{16}}$$

$$\left(\frac{49}{2}\right)^{16} \ge a_1 a_2 a_3 ... a_{16}$$

**30. (a, b) :** Let $r$ be common ratio of G.P., we have

$$z = \frac{\frac{a}{b} + i}{\frac{c}{b} - i} = \frac{\frac{1}{r} + i}{r - i} = \frac{i}{r} \Rightarrow z = \frac{ib}{c} \text{ or } \frac{ia}{b}$$

**31. (a, b, c) :** For any no. choosen from [1,180] there are 20 ways to select the second no. and from [181,199] there are 19, 18, …..1, ways resp. to select the second no. hence required no.of ways

$= 20 \times 180 + (19 + 18 + .......+1) = 3790$

**32. (a, b, d) :** If T comes in first toss then Mr. B can win in only one case that is TT.

Probability of Mr. B winning $= \frac{1}{4}$

Probability of Mr.A winning $= \frac{3}{4}$

Given first toss is head, Mr. A can win in successive tosses are T,HT, HHT, …..

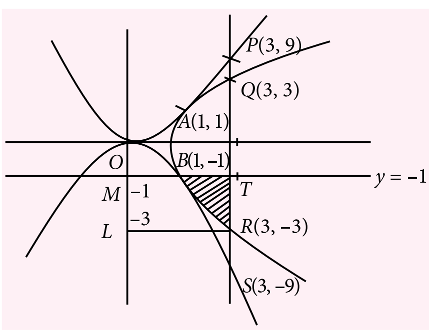
$$\text{Probability} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$$

Given first toss is head, Mr.A can win in the following cases HT, HHT, HHHT,…..

It is a G.P. with first term $= \frac{1}{4}$,

common ratio $= \frac{1}{2}$

$$\text{Probability} = \frac{\frac{1}{4}}{1-\frac{1}{2}} = \frac{1}{2}$$




**33. (d) :** Area $OAB$

$$= 2\int_0^1 \left(\frac{y^2+3}{4} - \sqrt{y}\right) dy = \frac{1}{3} \text{ sq. units}$$

**34. (a) :** Area $APQ = \int_1^3 (x^2 - \sqrt{4x-3}\; dx$ = 13/3 sq.units

**35. (a) :** Area $BTR$ = Area of rectangle $LMTR$

$$-\text{Area } LMBR = 6 - \int_{-3}^{-1} \frac{y^2+3}{4} dy = 7/3 \text{ sq. units}$$

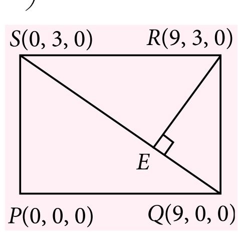
**36. (b) :** Equation of line $QS$ in 2-$D$ will be $x + 3y - 9 = 0$,

$RE = \frac{9}{\sqrt{10}}$ and $E \equiv \left(\frac{81}{10}, \frac{3}{10}, 0\right)$,

so point $T$ will be

$\left(\frac{81}{10}, \frac{3}{10}, \frac{9}{\sqrt{10}}\right)$,

Hence $PT = \frac{3}{5}\sqrt{205}$




**37. (c) :** Direction ratio of $QP \equiv 9, 0, 0$

Direction ratio of $QT \equiv \frac{9}{10}, \frac{-3}{10}, \frac{-9}{\sqrt{10}}$

So, $\cos\theta = \frac{3}{10} \Rightarrow \tan\theta = \frac{\sqrt{91}}{3}$

**38. (a) :** Shortest distance between the lines $\vec{r} = \vec{a} + \lambda\vec{\alpha}$ and $\vec{r} = \vec{b} + \mu\vec{\beta}$ is

given by $\frac{|(\vec{a}-\vec{b}).(\vec{\alpha}\times\vec{\beta})|}{|\vec{\alpha}\times\vec{\beta}|} = 3\sqrt{\frac{10}{19}}$

**39. (b) :** Take $B$ as origin, $BC$ as $x$-axis and take $A$ as $(h, k)$ $\therefore C(4,0)$.

Area of $\Delta ABC = \frac{1}{2}\times 4 \times k = 2k$ ...(i)

$D = (2,0)$ and $E\left(\frac{h+4}{2}, \frac{k}{2}\right)$

$\because$ $AD \perp BE$ $\therefore$ slope of $AD \times$ slope of $BE = -1$

$\Rightarrow$ $k^2 + (h + 4)(h - 2) = 0$ ...(ii)

Also $AC = 3$ $\Rightarrow$ $(h - 4)^2 + k^2 = 9$ ...(iii)

(ii)-(iii) gives

$\Rightarrow$ $h = \frac{3}{2}$ and $k^2 = \frac{11}{4}$

$\Rightarrow$ $k = \frac{\sqrt{11}}{2}$

From (i), Area of $\Delta ABC = \sqrt{11}$

**40. (c) :** Now $AD$ is the bisector $\frac{AB}{AC} = \frac{BD}{DC} \Rightarrow c = 2b$

$b + c > a \Rightarrow$ $b + c > 6$

$\therefore$ $b > 2$

Again $\frac{b^2+4b^2-3b}{4b^2} < 1$ $\Rightarrow b < 6$

$\therefore$ $2 < b < 6$ and consequently $4 < c < 12$

**41. (c) :** Now $c^2 = h^2 + k^2$ and $b^2 = (6 - h)^2 + k^2$

$c^2 - b^2 = 12h - 36 \Rightarrow h = \frac{b^2+12}{4}$

Given that $k^2 > 10$ $\Rightarrow$ $c^2 - h^2 > 10$

$\Rightarrow$ $4b^2 - \left(\frac{b^2+12}{4}\right)^2 > 10$

$\Rightarrow$ $b^2 \in (20-\sqrt{96}, 20+\sqrt{96})$

$B$ is either 4 or 5

**(42-44)**

Given $\frac{s-a}{s} = \frac{1}{2} \Rightarrow b + c = 3a \Rightarrow CA + BA = 12$

$\therefore$ $A$ lies on ellipse whose foci are $B$ and $C$

Centre of ellipse = (4, 4) and major axis parallel to $x$-axis

$\Rightarrow$ Length of major axis = 12 units

$\therefore$ $12e = 4 \Rightarrow e = \frac{1}{3}$

$\Rightarrow$ Length of minor axis = $8\sqrt{2}$ units

**42. (c) :** $PQ$ is the latus rectum of the ellipse.

$$\text{Area} = 2\left(\frac{1}{2}\times k_1 e \times \frac{k_2^2}{k_1}\right) = ek_2^2 = \frac{32}{3} \text{ sq. units}$$

**43. (d) :** $\Delta PBC$ is right angled at $C$.

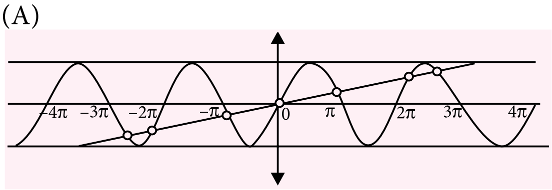
**44. (d) :** Equation of Hyperbola is

$$\frac{(x-4)^2}{4} - \frac{(y-4)^2}{4(e^2-1)} = 1$$

It passes through (0, 2) $\Rightarrow e = \frac{2}{\sqrt{3}}$

**45.** A $\to$ r; B $\to$ q; C $\to$ p; D $\to$ s

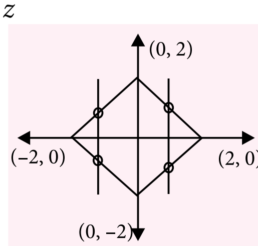
(A)




(B) $\frac{\pi x^2}{3} = (4n+1)\frac{\pi}{2},\ n \in z$

$x^2 = \frac{3}{2}(4n+1),\ n \in z$

$x = \pm\sqrt{\frac{3}{2}}$

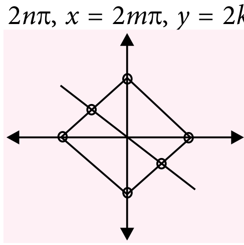



(C) $\sin\frac{\pi x}{2\sqrt{3}} = (x-\sqrt{3})^2 + 1$

$x = \sqrt{3}$

(D) $2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = 2\sin\frac{x+y}{2}.\cos\frac{x+y}{2}$

$x + y = 2n\pi,\ x = 2m\pi,\ y = 2k\pi$



**46.** A $\to$ r; B $\to$ p; C $\to$ s; D $\to$ q

(A) Let equation of tangent

$y = mx + \frac{2}{m} \Rightarrow 2 = -m + \frac{2}{m} \Rightarrow m^2 + 2m - 2 = 0$

$\therefore\ m_1 + m_2 = -2$

$m_1 m_2 = -2 \Rightarrow \frac{1}{m_1} + \frac{1}{m_2} - 1 = 0$

(B) $m_1 m_2 = -1 \Rightarrow b^2 = a^2 e^2 \Rightarrow 4e^2 = 2$

(C) $(y-2)^2 = -4(x+1)$ foot of $\perp$ will lie on tangent at vertex $x + 1 = 0 \Rightarrow k = 1$.

(D) $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1 \Rightarrow \frac{1}{e^2} = 1 - \frac{1}{4} \Rightarrow 3e^2 = 4$

**47.** A $\to$ s; B $\to$ q; C $\to$ p; D $\to$ p

(A) $n(s) = M^n\ n(A) = n! \Rightarrow P(A) = \frac{n!}{M^n}$

(B) $n(s) = {}^{(M+n-1)}C_{M-1}\ n(A) = 1$

$\Rightarrow P(A) = \frac{1}{{}^{M+n-1}C_{M-1}}$

(C) $n(s) = {}^M C_n\ n(A) = 1 \Rightarrow P(A) = \frac{1}{{}^M C_n}$

(D) $n(s) = {}^M C_n\ n!\ n(A) = n! \Rightarrow P(A) = \frac{1}{{}^M C_n}$

**48.** A $\to$ r; B $\to$ p; C $\to$ q; D $\to$ s

(A) The required event will occur if last digit in all the chosen numbers is 1, 3, 7 or 9.

$\therefore$ Required probability $= \left(\frac{4}{10}\right)^n$

(B) Required probability = $P$ (that the last digit is 2, 4, 6, 8)

= $P$ (that the last digit is 1, 2, 3, 4, 6, 7, 8, 9)

– $P$ (that the last digit is 1, 3, 7, 9) = $\frac{8^n - 4^n}{10^n}$.

(C) Rquired prob. = $P$ (1, 3, 5, 7, 9) – $P$(1, 3, 7, 9)

$= \frac{5^n - 4^n}{10^n}$

(D) Required prob. = $P$(0, 5) – $P$(5)

$= \frac{(10^n - 8^n) - (5^n - 4^n)}{10^n} = \frac{10^n - 8^n - 5^n + 4^n}{10^n}$

**49. (6) :** $\text{Area} = \left|\int_1^5 (6x - x^2 - 5)\,dx - \int_1^3 (4x - x^2 - 3)\,dx\right|$

$+ \left|\int_3^4 (4x - x^2 - 3)dx\right| + \left|\int_4^5 (3x - 15)dx\right| = \frac{73}{6}$

**50. (4) :** $\sum \frac{a}{r_1} = 2R \sum \frac{2\sin A/2 \cos A/2}{4R \sin A/2 \cos B/2 \cos C/2}$

$= \sum\left(\tan\frac{B}{2} + \tan\frac{C}{2}\right)$

$$= 2\sum \tan\frac{A}{2} = 2\sum\frac{r_1}{s} = 4\left(\frac{r_1+r_2+r_3}{a+b+c}\right)$$

**51. (1) :** $\left|\dfrac{1+x^2}{2x}\right| \le 1 \Rightarrow |x| = 1 \Rightarrow x = \pm 1$

But $x = -1$ will not satisfy the equation.

**52. (8) :** $x^2 + y^2 \le 144$ and $\sin(x + y) \ge 0$

$\Rightarrow$ $2n\pi \le x + y \le (2n + 1)\pi;\ n \in I$

Hence, we get the area

$S = \dfrac{\pi \cdot 144}{2} \Rightarrow \dfrac{S}{9\pi} = 8$

**53. (1) :** $[x] = \dfrac{5 \pm \sqrt{25 + 4\sin x - 24}}{2 \cdot 1} = \dfrac{5 \pm \sqrt{1 + 4\sin x}}{2}$

$-1 \le \sin x \le 1$

$\Rightarrow$ $-4 \le 4\sin x \le 4 \Rightarrow -3 \le 1 + 4\sin x \le 5$

$\Rightarrow$ $0 \le \sqrt{1 + 4\sin x} \le \sqrt{5}$

$\Rightarrow$ $[x]$ is an integer $\Leftrightarrow \sin x = 0$

$\Rightarrow$ $[x] = 3 \Rightarrow x = \pi$

**54. (5) :** Reflecting the point (1, 2, 3) in the $xy$-plane produces (1,2,–3) . A half turn about the $x$-axis yields (1, –2, 3). Finally translation 5 units will produce (1, 3, 3)

**55. (2) :** Equation of any plane; continuing the general plane is

$x + y + 2z - 3 + \lambda(2x + 3y + 4z - 4) = 0$ ...(i)

if plane (i) is parallel to $z$-axis $\Rightarrow \lambda = -\dfrac{1}{2}$

Therefore plane, parallel to $z$-axis is

$y + 2 = 0$ ...(ii)

Now, shortest distance between any point on $z$-axis (0, 0, $a$ ) (say) from plane (ii) is 2

**56. (9) :** The given curve represents the point $\left(-\dfrac{1}{2}, 2\right)$.

$\therefore$ Minimum distance = 1.

**57. (0) :** Combined equation of the tangents drawn from (0, 0)to the circle is

$(x^2 + y^2 - 2rx - 2hy + h^2)h^2 = (-rx - hy + h^2)^2$,

here coefficient of $x^2$ + coefficient of $y^2 = 0$

$\Rightarrow$ $(h^2 - r^2) + (h^2 - h^2) = 0$

$\Rightarrow$ $\dfrac{h}{r} = \pm 1$

**58. (1) :** Solving the equation of the lines, we get

$z = -\bar{z} \Rightarrow z = i$

$|\alpha - 1| = 2; \alpha = 2e^{i\theta} + i$, put it in the equation of the second line, we get $\cos\theta - \sin\theta = 0$

$\alpha = i \pm 2e^{\frac{i\pi}{4}}$ $\therefore$ $x = \pm\sqrt{2}$ $\Rightarrow$ $[|x|] = 1$

**59. (2) :** $\sin(2\sin x) = \sin\left(\dfrac{\pi}{2} - 2\cos x\right)$

$\sin x + \cos x = \dfrac{\pi}{4}$, implies

$1 + \sin 2x = \dfrac{\pi^2}{16}$

$\therefore$ $\tan x + \cot x = \dfrac{2}{\sin 2x} = \dfrac{2 \times 16}{\pi^2 - 16} = \dfrac{32}{\pi^2 - 16}$

$\therefore$ $a = 32,\ b = 16,\ c = 2$

So, $\dfrac{a + b + c}{25} = 2$

# MATHS MUSING

## SOLUTION SET-158

**1. (b) :** If $c = kb$, $k > 1$, then

$$b = \frac{4020kb}{2010 + kb} \Rightarrow k(4020 - b) = 2010$$

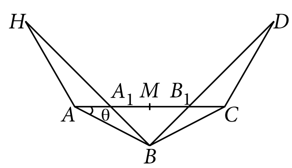
$\therefore$ $k$ is divisor of $2010 = 2\cdot 3\cdot 5\cdot 67$ $\therefore$ $N = 2^4 - 1 = 15$

**2. (c) :** $z = re^{i\theta}$

$$\left|z + \frac{1}{z}\right|^2 = r^2 + \frac{1}{r^2} + 2\cos 2\theta \Rightarrow a = \frac{3}{2}, b = \frac{17}{4}$$

$$\therefore \quad a + b = \frac{23}{4}$$

**3. (b) :** $\theta = \frac{\pi}{8}$




$A_1B_1 = 2A_1M$

$$= 2\left(\cos\theta - \frac{1}{2\cos\theta}\right)AB$$

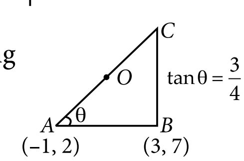
$$\frac{A_1B_1}{AB} = \frac{\cos 2\theta}{\cos\theta}, \left(\frac{A_1B_1}{AB}\right)^2 = \frac{1 + \cos 4\theta}{1 + \cos 2\theta} = 2 - \sqrt{2}$$

**4. (d) :** The given determinant is the product

$$\begin{vmatrix} 0 & 1 & 1 \\ 0 & a+b & c+d \\ 0 & ab & cd \end{vmatrix} \begin{vmatrix} 0 & 0 & 0 \\ 1 & c+d & cd \\ 1 & a+b & ab \end{vmatrix} = 0$$

**5. (a,c) :**

$O$ is the centre. Using rotations, we have

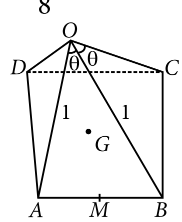



$$z = -1 + 2i \pm (4 + 5i)\frac{5}{8}cis\theta$$

$$= -1 + 2i \pm \frac{1}{8}(4 + 5i)(4 + 3i) = -\frac{7}{8} + 6i, \frac{23}{8} + 3i$$

$$d = |z_1| = \sqrt{\left(\frac{7}{8}\right)^2 + 36} \Rightarrow [d] = 6$$

$$d = |z_2| = \sqrt{\left(\frac{23}{8}\right)^2 + 9} \Rightarrow [d] = 4$$




$$\overline{OA} = \vec{a}, \overline{OB} = \vec{b}, \overline{OC} = \vec{c}, \overline{OD} = \vec{d}$$

$$AB = 2\sin\left(\frac{\theta}{2}\right), |\vec{a} - \vec{c}|^2 = 2AB^2 = 2 - 2\vec{a}\cdot\vec{c}$$

$$\therefore \quad \vec{a}\cdot\vec{c} = 1 - 4\sin^2\left(\frac{\theta}{2}\right) = 2\cos\theta - 1$$

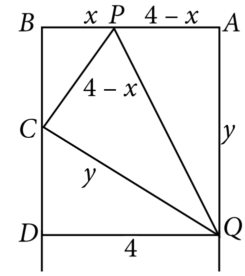
$$(\vec{a}\times\vec{b})\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} \cos\theta & 2\cos\theta - 1 \\ 1 & \cos\theta \end{vmatrix} = (1 - \cos\theta)^2$$

**6. (c) :** $x < \tan x$ and $\sin x/x$ dcreases $\Rightarrow h(x) > g(x)$
$\sin x < x$ and $\tan x/x$ increases $\Rightarrow h(x) > f(x)$

**7. (d) :** maximum between $a$ and $b$ and minimum between 1 and 2.

**8. (c) :** minimum between $a$ and $b$.

**9. (c) :** $A = \frac{\pi}{7}, B = \frac{2\pi}{7}, C = \frac{4\pi}{7}$




$\cos A = \cos B = \cos C = \frac{1}{8}$

$\sin A \sin B \sin C = \frac{\sqrt{7}}{8}$

$\therefore$ $\tan A + \tan B + \tan C = -\sqrt{7}$

$$\sum \tan A \tan B = -7$$

$$\Rightarrow \quad \sum \tan^2 A = 21, \sum \sec^2 A = 24$$

$$\sum \cot^2 A = 5, \sum \text{cosec}^2 A = 8 \quad \therefore \frac{\sum \sec^2 A}{\sum \text{cosec}^2 A} = 3$$

**10. (a) $\rightarrow$ (t); (b) $\rightarrow$ (r); (c) $\rightarrow$ (p); (d) $\rightarrow$ (q);**

(a) coeff. of $x^{10}$ in $(x + x^2 + \ldots + x^6)^4$

$$= \text{coeff. of } x^6 \text{ in } (1 - x^6)(1 - x)^{-4} = \binom{9}{3} - 4 = 80$$

$$\therefore \quad \text{Probability} = \frac{80}{6^4} = \frac{5}{81}$$

(b) Consider the 3 rows : 1, 4, 7, 10
2, 5, 8
3, 6, 9

$x$ and $y$ are from any of the 3 rows or one from the first and the other from the second row

$$\therefore \quad \text{Probability} = \frac{\binom{4}{2} + 2\binom{3}{2} + \binom{4}{1}\binom{3}{1}}{\binom{10}{2}} = \frac{8}{15}$$

(c) $\therefore$ $$\text{Probability} = \frac{\binom{10-3+1}{3}}{\binom{10}{3}} = \frac{\binom{8}{3}}{\binom{10}{3}} = \frac{7}{15}$$

(d) Number of triangles not having common side with the octagon is $\frac{8}{6}(8 - 4)(8 - 5) = 16$

### Solution Sender of Maths Musing

**SET-158**

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# PRACTICE PAPER

Exam date : 14th to 28th May 2016

# BITSAT

1. If $\arg(z) < 0$ then $\arg\left(\frac{\bar{z}-z}{2013}\right)$ is
   (a) $\pi/2$ (b) 0
   (c) $-\pi/2$ (d) $\pi$

2. If $a_1$, $a_2$, $a_3$, ...., $a_{2015}$ are in A.P. so that $\sum_{r=1}^{2014} \frac{1}{a_r a_{r+1}} = \frac{2014}{2013}$ and $a_{101} + a_{305} + a_{509} + a_{1507} + a_{1711} + a_{1915} = 6042$ then form the equation having roots $a_1$ and $a_{2015}$
   (a) $x^2 - 2013x + 2013 = 0$
   (b) $x^2 - 2015x + 2013 = 0$
   (c) $x^2 - 2014x + 2013 = 0$
   (d) none of these

3. Find the domain of $f(x) = \frac{1}{\sqrt[4]{\log_{1/3}\log_3\left(\frac{2x}{3+x}\right)}}$
   (a) $(-\infty, -9)$ (b) $(3, \infty)$
   (c) $(0, \infty)$ (d) none of these

4. Find the equation of tangent(s) to $3y^2 = 4x$ which passes through $\left(\frac{1}{2}, -\frac{1}{4}\right)$.
   (a) $3x + 2y = 1$ (b) $8x + 3y + 4 = 0$
   (c) $x - 2y = 1$ (d) no tangent form

5. If $f(x) = \cos x$, $g(x) = \sqrt{x}$ then $\lim_{x\to\infty} (fog(x+1) - fog(x)) =$
   (a) 1 (b) 0
   (c) does not exist (d) none of these

6. How many integral solution exists in $[-2014, 4319]$ of the equation $[|210 - x|] = 4321 - [|x - 85|]$ ? ( $[\cdot]$ denotes greatest integer function)
   (a) 210 (b) 4321
   (c) 85 (d) none of these

7. If tangent and normal to $x^2 + 4y^2 = 9$ at $\theta = \frac{\pi}{4}$ be $x + \alpha y = \beta$ and $\alpha x - y = \gamma$ respectively, then which of the following are in G.P.?
   (a) $\alpha, \beta, \gamma$ (b) $\alpha, \sqrt{2}\beta, 2\gamma$
   (c) $\alpha, \beta, 2\sqrt{2}\gamma$ (d) none of these

8. Find the period of $\sum_{r=1}^{2013} \sin 2r\pi\{x\}$ (where $\{\cdot\}$ denotes fractional part of $x$)
   (a) $\pi$ (b) 2013
   (c) 1 (d) none of these

9. Find the number of solution of system of equations $x + \frac{2}{3}y + \frac{z}{3} = 0$, $y + 2z = 0$ and $x + \frac{4}{3}y + \frac{5}{3}z = 0$
   (a) no solution
   (b) only trivial solution
   (c) infinite non-trivial solution
   (d) none of these

10. If $\vec{a}$ and $\vec{b}$ satisfy $\vec{a} + \vec{b} = \vec{p}$, $2\vec{a} + \vec{b} = \vec{q}$ (where $\vec{p} = 2\hat{i} - 3\hat{j}$, $\vec{q} = -\hat{i} - \hat{j}$) and angle between $\vec{a}$ and $\vec{b}$ is $\theta$ such that $\cos\theta + \sqrt{\frac{\alpha}{\alpha+1}} = 0$, then find $\alpha$.
    (a) 5 (b) 13
    (c) 26 (d) none of these

11. $\int \frac{\sqrt{x} + \frac{3}{x\sqrt{x}} + \frac{5}{x^3\sqrt{x}}}{\sqrt{x^5 - x^4 - x^2 - 1}}\, dx =$
    (a) $\frac{2}{x^4}\sqrt{x^4 - x^3 - x - \frac{1}{x}} + C$
    (b) $2\sqrt{1 + x + x^4 + x^5} + C$
    (c) $2\sqrt{x^4 - x^3 - x - \frac{1}{x}} + C$
    (d) none of these

**12.** If $P(A) = x$, $P(B) = y$ $(\neq 0)$ then $P(A/B) \in$

(a) $\left[\frac{x-1}{y}, \frac{x}{y}\right]$ (b) $\left[\frac{x+y-1}{y}, \frac{x+y}{y}\right]$

(c) $\left[\frac{x}{y}, \frac{x+y}{y}\right]$ (d) $\left[\frac{x+y-1}{y}, \frac{1}{y}\right]$

**13.** If from $(\alpha, \beta)$, two tangents are drawn to $y^2 = 4x$ so that slopes of tangents are in the ratio 1 : 2 and

$f(x) = \frac{\alpha x^2}{1} + \frac{\beta x}{2} + \frac{1}{3}$, then

(a) $f(x) > 0$

(b) $\alpha$ can't be negative

(c) Locus of $(\alpha, \beta)$ is a parabola

(d) all of these

**14.** $\cos^{-1}\left(\cos\left(12\cot^{-1}\left(\frac{\sqrt{8\sqrt{5}\cos\frac{\pi}{5}}}{4\sin\frac{\pi}{10}}\right)\right)\right) =$

(a) $\frac{6\pi}{5}$ (b) $\frac{\pi}{5}$

(c) $\frac{4\pi}{5}$ (d) $-\frac{4\pi}{5}$

**15.** If $A = \int_0^{504\pi} |\cos x|\, dx, B = \int_{504\pi}^{1007\pi} |\sin x|\, dx$,

then which of the following is equal to 2013?

(a) $A + B$ (b) $A + B - 1$

(c) $\frac{1}{2}(A + B - 1)$ (d) $2(A + B - 1)$

**16.** Find the number of solution of

$3 + [x] = \log_2 (9 - 2^{\{x\}}) + x$ in $[-1, 4]$.

(a) 6 (b) 12

(c) 2 (d) None of these

**17.** If $\alpha, \beta, \gamma$ be the roots of $x^3 + (a^4 + 4a^2 + 1)x = x^2 + a^2$,

then minimum value of $\sum\left\{\frac{\alpha}{\beta} + \left(\frac{\alpha}{\beta}\right)^{-1}\right\}$ is

(a) 6 (b) 8

(c) 4 (d) None of these

**18.** $S$ and $S'$ be foci of $16x^2 + 25y^2 = 400$ and $P\left(\frac{5}{2}, 2\sqrt{3}\right)$ is a point on it. Normal at $P$ meets $x$-axis at $A$ then $SA/S'A =$

(a) $\frac{15}{8}$ (b) $\frac{7}{13}$

(c) $\frac{13}{7}$ (d) $\frac{7}{13}$ or $\frac{13}{7}$

**19.** If $(x^{2012} + 2 + x^{2014})^{2013}$ be expanded in ascending powers of $x$, in the form $P_0 + P_1x + P_2x^2 + P_3x^3 + \ldots$, then

(a) $2(P_0 + P_3 + P_6\ldots\ldots) = (P_1 + P_2 + P_4 + P_5\ldots\ldots) + 2$

(b) $(P_0 + P_3 + P_6\ldots\ldots) = (P_1 + P_2 + P_4 + P_5\ldots\ldots) + 1$

(c) $2(P_0 + P_3 + P_6\ldots\ldots) = (P_1 + P_2 + P_4 + P_5\ldots\ldots)$

(d) None of these

**20.** The number of solution of $\tan^{-1}(x + 2) +$

$\cot^{-1}\sqrt{4x+20} = \lim_{x \to \frac{\pi}{2}} \frac{\sin(x\cos x)}{\cos(x\sin x)}$ in $[-5, 5]$ is

(a) 2 (b) 1

(c) 4 (d) None of these

**21.** If $f(x) = (1 + [x])^{\frac{1}{x - \{x\}}}$ (where $[x]$ & $\{x\}$ denote the integral part & fractional part of $x$)

and $a = \lim_{x \to 0^-} f(x)$, $b = \lim_{x \to 0^+} f(x)$

and $c = \lim_{x \to 0} f(x)$ then

(a) only $a$ exists

(b) only $b$ exists

(c) $a$, $b$, $c$ all exist

(d) $a$ & $b$ exists but $c$ does not exist

**22.** If imaginary part of $(1 - i)^n(1 + i)^{-n}$ be negative (where $n \in N$, $n < 100$) then sum of all the possible values of $n$ is

(a) 1128 (b) 625

(c) 1225 (d) None of these

**23.** If $R \subseteq A \times B$ and $S \subseteq B \times C$ be two relations, then $(SoR)^{-1}$ is equal to

(a) $S^{-1}\, oR^{-1}$

(b) $RoS$

(c) $R^{-1}oS^{-1}$

(d) none of these

**24.** The region of Argand diagram defined by

$|z - 1| + |z + 1| \leq 4$ is

(a) interior of an ellipse

(b) exterior of a circle

(c) interior and boundary of an ellipse

(d) none of these

25. If $f(x)=\sqrt{3|x|-x-2}$ and $g(x)$ = sin $x$, then domain of definition of $fog\,(x)$ is

(a) $\left\{2n\pi+\frac{\pi}{2}\right\}, n\in I$

(b) $\bigcup_{n\in I}\left(2n\pi+\frac{7\pi}{6}, 2n\pi+\frac{11\pi}{6}\right)$

(c) $\left(2n\pi+\frac{7\pi}{6}\right), n\in I$

(d) $\{(4m+1)\frac{\pi}{2} : m\in I\}\bigcup_{n\in I}\left(2n\pi+\frac{7\pi}{6}, 2n\pi+\frac{11\pi}{6}\right)$

26. Equation of the plane which passes through the line of intersection of the planes $P = ax + by + cz + d = 0$, $P' = a'x + b'y + c'z + d' = 0$ and parallel to $x$-axis, is

(a) $Pa - P'a' = 0$ (b) $P/a = P'/a' = 0$

(c) $Pa + P'a' = 0$ (d) $P/a = P'/a'$

27. The mid points of the sides of a triangle are (5, 0), (5, 12) and (0, 12). The orthocentre of this triangle is

(a) (0, 0) (b) (10, 0)

(c) (0, 24) (d) (13/3, 8)

28. The function $f(x)=\int_0^x \log(t+\sqrt{1+t^2})\,dt$ is

(a) an even function (b) an odd function

(c) a periodic function (d) none of these

29. In a $\Delta ABC$, the angles $A$ and $B$ are two values of $\theta$ satisfying $\sqrt{3}\cos\theta+\sin\theta=k, |k|<2$. The triangle

(a) is a acute angled (b) is a right angled

(c) is an obtuse angled (d) has one angle $=\frac{\pi}{3}$

30. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and $\lambda$ is a real number then $[\lambda(\vec{a}+\vec{b})\ \ \lambda^2\vec{b}\ \ \lambda\vec{c}]=[\vec{a}\ \ \vec{b}+\vec{c}\ \ \vec{b}]$ for

(a) exactly three values of $\lambda$

(b) exactly two values of $\lambda$

(c) exactly one value of $\lambda$

(d) no real value of $\lambda$

31. The statement $P(n) : 1\times 1! + 2\times 2! + 3\times 3! + \ldots + n\times n! = (n+1)! - 1$ is

(a) true for all $n > 1$ (b) true for no $n$

(c) true for all $n \in N$ (d) none of these

32. The vector $\vec{c}$ directed along the internal bisector of the angle between the vectors $\vec{a}=7\hat{i}-4\hat{j}-4\hat{k}$ and $\vec{b}=-2\hat{i}-\hat{j}+2\hat{k}$ with $|\vec{c}|=5\sqrt{6}$ is

(a) $\pm\frac{5}{3}(\hat{i}-7\hat{j}+2\hat{k})$ (b) $\frac{5}{3}(5\hat{i}+5\hat{j}+2\hat{k})$

(c) $\frac{5}{3}(\hat{i}+7\hat{j}+2\hat{k})$ (d) $\frac{5}{3}(-5\hat{i}+5\hat{j}+2\hat{k})$

33. The equation of the plane through the points (2, 2, 1) and (9, 3, 6) and perpendicular to the plane $2x + 6y + 6z - 1 = 0$ is

(a) $3x + 4y + 5z = 9$

(b) $3x + 4y - 5z = 9$

(c) $3x + 4y - 5z + 9 = 0$

(d) none of these

34. Area of the region bounded by the curves $y = 2^x$, $y = 2x - x^2$, $x = 0$ and $x = 2$ is given by

(a) $\frac{3}{\log 2}-\frac{4}{3}$ (b) $\frac{3}{\log 2}+\frac{4}{3}$

(c) $3\log 2-\frac{4}{3}$ (d) $3\log 2+\frac{4}{3}$

35. If $\bar{X}_1$ and $\bar{X}_2$ are the means of two distributions such that $\bar{X}_1 < \bar{X}_2$ and $\bar{X}$ is the mean of the combined distribution, then

(a) $\bar{X} < \bar{X}_1$ (b) $\bar{X} > \bar{X}_2$

(c) $\bar{X}=\frac{\bar{X}_1+\bar{X}_2}{2}$ (d) $\bar{X}_1 < \bar{X} < \bar{X}_2$

36. A straight line through the point $(h, k)$ where $h > 0$ and $k > 0$, makes positive intercepts on the coordinate axes. Then the minimum length of the line intercepted between the coordinate axes is

(a) $(h^{2/3} + k^{2/3})^{3/2}$ (b) $(h^{3/2} + k^{3/2})^{2/3}$

(c) $(h^{2/3} - k^{2/3})^{3/2}$ (d) $(h^{3/2} - k^{3/2})^{2/3}$

37. The sum of the rational terms in the expansion of $(\sqrt{2}+\sqrt[5]{3})^{10}$ is

(a) 41 (b) 42

(c) 39 (d) 45

38. Let $f(x)=\begin{cases} x^n \sin\frac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$

Then $f(x)$ is continuous but not differentiable at $x = 0$ if

(a) $n \in (0, 1]$ (b) $n \in [1, \infty)$

(c) $n \in (-\infty, 0)$ (d) $n = 0$

39. The objective function $Z = 4x + 3y$ can be maximized subjected to the constraints $3x + 4y \le 24$, $8x + 6y \le 48$, $x \le 5$, $y \le 6$; $x, y \ge 0$.

(a) at only one point
(b) at two points only
(c) at an infinite number of points
(d) none of these

**40.** If $\int \frac{1}{(\sin x+4)(\sin x-1)}dx$

$$= A\frac{1}{\tan\frac{x}{2}-1}+B\tan^{-1}(f(x))+C, \text{ then}$$

(a) $A=\frac{1}{5}, B=\frac{-2}{5\sqrt{15}}, f(x)=\frac{4\tan x+3}{\sqrt{15}}$

(b) $A=-\frac{1}{5}, B=\frac{1}{\sqrt{15}}, f(x)=\frac{4\tan\left(\frac{x}{2}\right)+1}{\sqrt{15}}$

(c) $A=\frac{2}{5}, B=\frac{-2}{5}, f(x)=\frac{4\tan x+1}{5}$

(d) $A=\frac{2}{5},\ B=\frac{-2}{5\sqrt{15}}, f(x)=\frac{4\tan\frac{x}{2}+1}{\sqrt{15}}$

**41.** In a moderately skewed distribution the values of mean and median are 5 and 6 respectively. The value of mode in such a situation is approximately equal to
(a) 8 (b) 11
(c) 16 (d) none of these

**42.** The equation of the one of the tangents to the curve $y = \cos(x + y)$, $-2\pi \le x \le 2\pi$ that is parallel to the line $x + 2y = 0$, is

(a) $x + 2y = 1$ (b) $x+2y=\frac{\pi}{2}$

(c) $x+2y=\frac{\pi}{4}$ (d) none of these

**43.** Consider the integrals

$$I_1=\int_0^1 e^{-x}\cos^2 x\,dx,\ I_2=\int_0^1 e^{-x^2}\cos^2 x\,dx,$$

$$I_3=\int_0^1 e^{-x^2}dx \text{ and } I_4=\int_0^1 e^{-(1/2)x^2}dx.$$

The greatest of these integrals is
(a) $I_1$ (b) $I_2$
(c) $I_3$ (d) $I_4$.

**44.** Between two junction stations $A$ and $B$ there are 12 intermediate stations. The number of ways in which a train can be made to stop at 4 of these stations so that no two of these halting stations are consecutive is
(a) $^8C_4$ (b) $^9C_4$
(c) $^{12}C_4-4$ (d) none of these

**45.** $AB$ is a vertical pole. The end $A$ is on the level ground. $C$ is the middle point of $AB$. $P$ is a point on the level ground. The portion $BC$ subtends an angle $\beta$ at $P$. If $AP = n\,AB$, then $\tan\beta =$

(a) $\frac{n}{2n^2+1}$ (b) $\frac{n}{n^2-1}$

(c) $\frac{n}{n^2+1}$ (d) none of these

## ANSWER KEYS

| | | | | |
|---|---|---|---|---|
| 1. (a) | 2. (c) | 3. (d) | 5. (b) | 6. (d) |
| 7. (c) | 8. (c) | 9. (c) | 10. (d) | 11. (d) |
| 12. (b) | 13. (d) | 14. (c) | 15. (b) | 16. (a) |
| 17. (d) | 18. (d) | 19. (a) | 21. (b) | 22. (c) |
| 23. (c) | 24. (c) | 25. (d) | 26. (d) | 27. (a) |
| 28. (a) | 29. (c) | 30. (d) | 31. (c) | 32. (a) |
| 33. (b) | 34. (a) | 35. (d) | 36. (a) | 37. (a) |
| 38. (a) | 39. (c) | 40. (d) | 41. (a) | 42. (b) |
| 43. (d) | 44. (b) | 45. (a) | | |

# YOU ASK WE ANSWER

**Do you have a question that you just can't get answered?**

Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.

The best questions and their solutions will be printed in this column each month.

**1.** Find the domain of function

$$f(x)=\sqrt{\log_{0.4}\left(\frac{x-1}{x+5}\right)}+\frac{1}{x^2-36}.$$

***– Chinmay, Assam***

**Ans.** $f(x)=\sqrt{\log_{0.4}\left(\frac{x-1}{x+5}\right)}+\frac{1}{x^2-36} \Rightarrow f(x)=f_1+f_1$

For domain of $f_1$, $f_1=\sqrt{\log_{0.4}\left(\frac{x-1}{x+5}\right)}$

It is defined, if $\log_{0.4}\left(\frac{x-1}{x+5}\right)\geq 0 \Rightarrow \left(\frac{x-1}{x-5}\right)\leq 1$

and $\frac{x-1}{x+5}>0 \;\Rightarrow\; 0<\frac{x-1}{x+5}\leq 1 \;\Rightarrow\; \frac{x-1}{x+5}>0$

$\Rightarrow\; x\in(-\infty,-5)\cup(1,\infty)$ and $\frac{x-1}{x+5}-1\leq 0$

$\Rightarrow\; \frac{-6}{x+5}\leq 0 \Rightarrow \frac{6}{x+5}\geq 0 \;\Rightarrow\; x+5\geq 0 \Rightarrow x\geq -5$

$\therefore$ Domain of $f_1=(1,\infty)$

For domain of $f_2$, $f_2=\frac{1}{x^2-36}$

$x^2-36\neq 0 \Rightarrow x\neq \pm 6$

Hence, domain of $f(x)=(1,\infty)-\{6,-6\}$

$=(1,6)\cup(6,\infty)$

**2.** Find the angle of intersection of curves, $y=[|\sin x|+|\cos x|]$ and $x^2+y^2=5$, where $[\cdot]$ denotes greatest integral function.

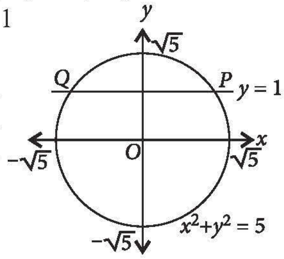
***– Nandini, Gujarat***

**Ans.** We know that, $1\leq|\sin x|+|\cos x|\leq\sqrt{2}$

$\Rightarrow y=[|\sin x|+|\cos x|]=1$

Let $P$ and $Q$ be the points of intersection of given curves. Clearly, the given curves meet at points where $y=1$, so we get

$x^2+1=5 \;\Rightarrow\; x=\pm 2$




Now, $P(2,1)$ and $Q(-2,1)$

On differentiating $x^2+y^2=5$ w.r.t. $x$, we get

$$2x+2y\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=-\frac{x}{y}$$

$$\left(\frac{dy}{dx}\right)_{(2,1)}=-2 \text{ and } \left(\frac{dy}{dx}\right)_{(-2,1)}=2$$

Clearly, the slope of line $y=1$ is zero and the slope of the tangents at $P$ and $Q$ are $(-2)$ and $(2)$, respectively.

Thus, the angle of intersection is $\tan^{-1}(2)$.

**3.** The area contained between the curve $y=\frac{a^3}{x^2+a^2}$ and $x$-axis is

***– Raman, A.P.***

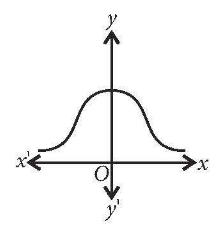
**Ans.** Since, the curve is symmetrical about $y$-axis.

$\therefore$ Required area $=2\int_0^\infty y\,dx$

$=2a^3\int_0^\infty\frac{dx}{x^2+a^2}$

$=2a^2\left[\tan^{-1}\frac{x}{a}\right]_0^\infty$

$=\pi a^2$ sq. units



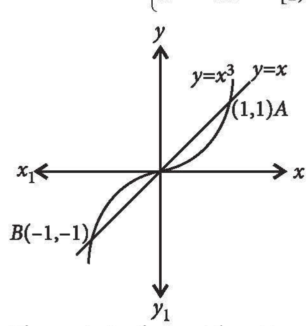

**4.** Let $f: R\to R$ be a function defined by $f(x)=\max(x,x^3)$. The set of all points where $f(x)$ is not differentiable, is

***– Raman, A.P.***

**Ans.** $f(x)=\max\{x,x^3\}$. Considering the graph separately.

$y=x^3$ and $y=x$

$$\text{Now, } f(x)=\begin{cases} x & \text{in } (-\infty,-1)\\ x^3 & \text{in } [-1,0)\\ x & \text{in } [0,1)\\ x^3 & \text{in } [1,\infty)\end{cases}$$




The point of consideration are

$f'(-1^-)=1$ and $f'(-1^+)=3$

$f'(-0^-)=0$ and $f'(-0^+)=1$

$f'(1^-)=1$ and $f'(1^+)=3$

Hence, $f$ is not differentiable at $-1, 0, 1$.